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Stochastic Behaviour of Single Unit System with Preventive Maintenance subject to Random Appearance of Server

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***Abstract* —** In this paper using Semi-Markovian approach a reliability model of single unit is develop. Unit works in operative mode initially and goes for preventive maintenance after a pre-specific time of operation. Single repairperson that may be appear and disappear randomly does every repair work including preventive maintenance. New one replaces the failed unit in case its repair is not possible by the server in a given fixed repair time. The repair activities and repairperson (server) are perfect. The random variables associated with failure time of the unit and different repair activities are independent to each other. The failure time and maximum operation time of the unit are exponentially distributed while the distributions of time of appearance and disappearance of server, repair and replacement of the unit are taken as arbitrary with different probability density functions. Applying regenerative point techniques, different reliability attributes are obtained to enhance the performance of system. Numerical results of Mean time to system failure (MTSF), Availability and profit function that are very much helpful to system engineer have also been analysed.

Keywords — Single Unit, Random Appearance, Reliability, Preventive Maintenance

I. Introduction

The automation and ever-increasing demand of society generates multifarious design systems. The proper utilization and maintenance of such systems becomes a big challenge. For this many industries adopted redundancy technique to improve their performance and availability. However, a unit cannot be kept as spare in view of their practical utility and common man's affordability. Thus, to improve the performance, a large number of researchers suggested single-unit systems with immediate arrival of server subject to different sets of assumptions and ideas such as unrepairable spare parts (Nakagawa and Osaki, 1976) , preventive maintenance (Abdul Ameer and Murari,1988), different failure modes (Chander and Bansal, 2005; Pawar and Malik, 2011), online repair and replacement (Chander,2007), different weather conditions (Malik and Barak, 2009; Kadyan and Promila,2012), etc. However, the expectation that every time server visits the system immediately when required by these authors seems to be impracticable. Consequently the single unit system with vacation of server (Renbin and Zaiming,2011), two-unit cold standby system with random appearance and disappearance time of server (Singh, 1989), two unit parallel system with maximum repair time and random appearance of server (Malik and Gitanjali,2012) were introduced.

Hence to maintain a required level of reliability and system performance, here a single-unit system with the concepts of preventive maintenance, random appearance of server is analyzed. Unit works in operative mode initially and goes for preventive maintenance after a pre-specific time of operation. Single repairperson that may be appear and disappear randomly does every repair work including preventive maintenance. New one replaces the failed unit in case its repair is not possible by the server in a given fixed repair time. The repair activities and repairperson (server) are perfect. The random variables associated with failure time of the unit and different repair activities are independent to each other. The failure time and maximum operation time of the unit are exponentially distributed while the distributions of time of appearance and disappearance of server, repair and replacement of the unit are taken as arbitrary with different probability density functions. Applying regenerative point techniques, different reliability attributes are obtained to enhance the performance of system. Numerical results of Mean time to system failure (MTSF), Availability and profit function that are very much helpful to system engineer have also been analysed.

The work in this paper is organized by labelling the introduction part in section 1 followed by the state transition diagram and its notations in section 2. Mathematical formulation of transition probabilities and various reliability measures such as MTSF, Availability, Busy period analysis of server etc has been done in section 3. Section 4 describes the cost-benefit analysis of reliability measures followed by the case study (numerical analysis) in section 5. Section 6 consists the discussion about the proposed work with the help of tabular data followed by conclusion in Section 7.

II. NOTATIONS

E	:	Set of regenerative states.
O	:	Unit is operative.
α_0	:	Maximum constant rate of repair time of unit
η	:	Maximum constant rate of operation time of unit
λ	:	Constant failure rate of the unit
NA/A	:	Server is not available / available
a/b	:	Constant rate of appearance / disappearance of server
$f(t)/F(t)$	:	pdf / cdf of the replacement time of the unit
$g(t)/G(t)$	:	pdf / cdf of the repair time of the unit
$p(t)/P(t)$	:	pdf / cdf of the maintenance time of the unit.
FW_r / FW_R	:	Unit is failed and waiting for repair/waiting for repair continuously from previous state.
FU_r / FU_R	:	Unit is failed and under repair / under repair continuously from previous state.
W_{pm} / W_{PM}	:	Unit is waiting for preventive maintenance / waiting for preventive maintenance continuously from previous state
U_{pm} / U_{PM}	:	Unit is under preventive maintenance / under preventive maintenance continuously from previous state
FU_{Rp} / FU_{RP}	:	Unit is failed and under replacement with ordinary server / under replacement continuously from previous state with ordinary server.
qij (t)/ Qij(t)	:	pdf / cdf of passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0, t].
$\sim/*$	:	Symbol for Laplace Stieltjes transform / Laplace transform.
®/ ©	:	Symbols for Stieltjes convolution Laplace convolution.

The possible transitions between states along with transitions rates for the system model are shown in Figure 1. All states (S0–S6) are regenerative.

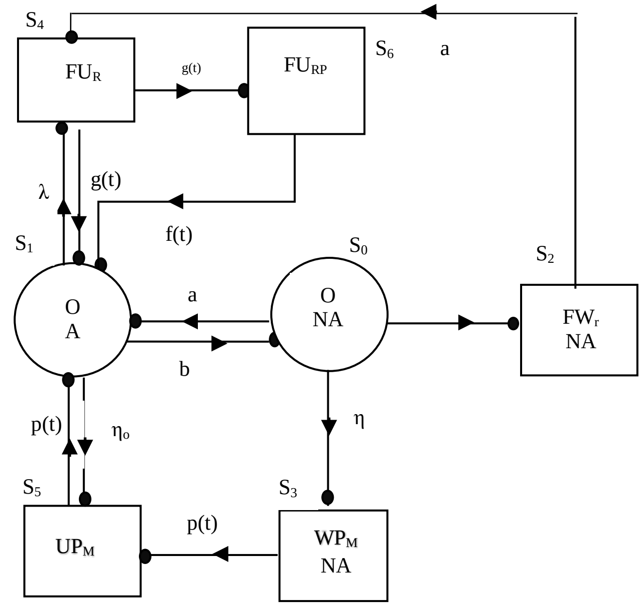

Figure1: State Transition Diagram

: Regenerative Point : Failed State O : Upstate

III. TRANSITION PROBABILITIES AND RELIABILITY MEASURES

The transition probability matrix (t.p.m) of the embedded Markov Chain is $p = p_{ij} = Q_{ij}(\infty) = Q(\infty)$. By Probabilistic arguments, the non-zero elements p_{ij} are

$$p_{01} = \frac{a}{a+\lambda+\eta}, \quad p_{02} = \frac{\lambda}{a+\lambda+\eta}, \quad p_{03} = \frac{\eta}{b+\lambda+\eta},$$
$$p_{10} = \frac{b}{b+\lambda+\eta}, \; p_{14} = \frac{\lambda}{a+\lambda+\eta}, \; p_{41} = \frac{\theta}{\theta+\alpha_0},$$
$$p_{46} = \frac{\alpha_0}{\theta+\alpha_0} \quad (1)$$

It can easily be verified that

$$p_{01} + p_{02} + p_{03} = p_{10} + p_{14} + p_{15} = p_{24} = p_{35} = p_{41} + p_{46} = p_{51} = 1 \quad (2)$$

MEAN SOJOURN TIMES

The mean sojourn times μ_i in state S_i is given by

$$u_0 = m_{01} + m_{02} + m_{03} = \frac{1}{a+\lambda+\eta},$$

$$u_1 = m_{10} + m_{14} + m_{15} = \frac{1}{b+\lambda+\eta}, \qquad u_2 = m_{24} = \frac{1}{a},$$

$$u_3 = m_{35} = \frac{1}{\rho}, \qquad u_1 = m_{41} + m_{47} = \frac{1}{\theta+\alpha_0},$$

$$u_5 = m_{51} = \frac{1}{\rho}, \qquad u_6 = m_{61} = \frac{1}{\beta}, \qquad (3)$$

Where $m_{ij} = E(T_{ij}) = \int_0^\infty t d\left[Q_{ij}(t)\right] = -q_{ij}^*(0)$ is the unconditional mean time taken by the system to transit from any state S_i when time is counted from epoch at entrance into state S_j.

Reliability and Mean time to system failure (MTSF)

Let $\emptyset_i(t)$ be the cdf of the first passage time from regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relation for $\emptyset_i(t)$:

$$\varphi_i(t) = \sum_j Q_{i,jj}(t) ® \varphi_j(t) + \sum_k Q_{i,jk}(t) \qquad (4)$$

Where Sj is an un-failed regenerative state to which the given regenerative state Si can transit and k is a failed state to which the state I can transit directly. Taking $L.S.T$ of relation (4) and solving for $\varphi_0^{**}(s)$, we get

$$R^*(s) = \frac{1-\varphi_0^{**}(s)}{s} \qquad (5)$$

The reliability R(t) can be obtained by taking inverse Laplace transformation of (5) and MTSF is given by

$$MTSF(T_0) = \lim_{s\to 0} R^*(s) = \lim_{s\to 0} \frac{1-\varphi_0^{**}(s)}{s} = \frac{N_1}{D_1} \qquad (6)$$

Where $N_1 = u_0 + p_{01}u_1$ and $D_1 = 1 - p_{01}p_{10}$.

Availability analysis

Let $A_i(t)$ be the probability that the system is in upstate at instant t given that the system entered regenerative state i at $t=0$.The recursive relation for $A_i(t)$ are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^n(t) © A_j(t) \qquad (7)$$

Where j is any successive regenerative state to which the regenerative state i can transit through n≥1 (natural number) transitions and Mi(t) is the probability that the system is up initially in regenerative state Si∈E at time $'t`$ without visiting to any other regenerative state. We have $M_o(t) = e^{-(\lambda+a)t}$ and $M_1(t) = e^{-(\lambda+b+\eta)t}$

Taking $L.T.$ of relations (7) and solving for $A_0^*(s)$, we get steady-state availability as

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2} \qquad (8)$$

Where $N_2 = u_1 + p_{10}u_0$ and

$$D_2 = u_1 + p_{10}u_0 + p_{02}u_2 + p_{03}p_{10}u_3 + (p_{02}p_{10} + p_{14})(u_4 + p_{46}u_6) + (p_{03}p_{10} + p_{15})u_5$$

Busy Period Analysis of server

Let $B_i^r(t)$ be the probability that the server is busy due to preventive maintenance, repair and replacement at an instant 't' given that the system entered regenerative state S_i at $t=0$. The following are the recursive relations for $B_i^r(t)$ are given as

$$B_i^r(t) = W_i(t) + \sum_j q_{i,j}^n(t) © B_j^r(t) \qquad (9)$$

where j is a subsequent regenerative state to which state i transits through n≥1(natural number) transitions and

$$W_4(t) = e^{-\alpha_0 t}\overline{G(t)} + \left(\alpha_0 e^{-\alpha_0 t} © 1\right)\overline{G(t)},$$

$$W_5(t) = \overline{P(t)} \quad \text{and} \quad W_6(t) = \overline{F(t)} \qquad (10)$$

Taking $L.T.$ of relation (9) and solving for $B_0^{r*}(s)$, we get in the long run the time for which the server is busy in steady state given by

$$B_0^r = \lim_{s\to 0} sB_0^{r*}(s) = \frac{N_3}{D_2} \qquad (11)$$

Where

$N_3 = (p_{14} + p_{02}p_{10})\left(W_6^*(0)p_{46} + W_4^*(0)\right) + (p_{15} + p_{03}p_{10})W_5^*(0)$ and D_2 is already specified.

Expected number of visits by the server

Let be the expected number of visits by the server in $(0, t]$ given that the system entered the regenerative state S_i at $t=0$. The recursive relation for $N_i(t)$ are given by

$$N_i(t) = \sum_j Q_{i,j}(t) ® \left[\delta_j + N_j(t)\right] \qquad (12)$$

Where j is any regenerative state to which the given regenerative state i transits and δ_i=1, if j is the regenerative state where the server does job afresh, otherwise δ_i= 0. Taking $L.S.T.$ of relation (12) and solving for $N_0^{**}(s)$, we get the expected number of visits by ordinary server per unit time as

$$N_0 = \lim_{s\to 0} sN_0^{**}(s) = \frac{N_4}{D_2} \quad (13)$$

Where $N_4 = \left(p_{10}\left(p_{01}+p_{02}p_{24}+p_{03}p_{35}\right)\right)$ and D_2 is already specified.

Expected number of preventive maintenance by the server

Let $N_i^{pm}(t)$ be the expected number of preventive maintenance by server $(0,t]$ given that the system entered the regenerative state S_i at $t=0$. The recursive relation for $N_i^{pm}(t)$ are given by

$$N_i^{pm}(t) = \sum_j Q_{i,j}(t) \circledR \left[\delta_j + N_j^{pm}(t)\right] \quad (14)$$

Where j is any regenerative state to which the given regenerative state i transits and δ_i=1, if j is the regenerative state where the expert server does job afresh, otherwise δ_i= 0. Taking $L.S.T.$of relation(14) and solving for $N_0^{pm**}(s)$, we get the expected number of visits by expert server per unit time as

$$N_0^{pm} = \lim_{s\to 0} sN_i^{pm**}(s) = \frac{N_5}{D_2} \quad (15)$$

Where $N_5 = \left(p_{15}+p_{03}p_{10}\right)$ and D_2 is already specified.

Expected number of repair by the server

Let $N_i^r(t)$ be the expected number of preventive maintenance by server $(0,t]$ given that the system entered the regenerative state S_i at $t=0$. The recursive relation for $N_i^r(t)$ are given by

$$N_i^r(t) = \sum_j Q_{i,j}(t) \circledR \left[\delta_j + N_j^r(t)\right] \quad (16)$$

Where j is any regenerative state to which the given regenerative state i transits and δ_i=1, if j is the regenerative state where the expert server does job afresh, otherwise δ_i= 0. Taking $L.S.T.$of relation(16) and solving for $N_0^{r**}(s)$, we get the expected number of visits by expert server per unit time as

$$N_0^r = \lim_{s\to 0} sN_i^{r**}(s) = \frac{N_6}{D_2} \quad (17)$$

Where $N_6 = \left(p_{41}\left(p_{14}+p_{02}p_{14}\right)\right)$ and D_2 is already specified.

Expected number of replacements of the unit

Let $R_i(t)$ be the expected number of replacements by the unit in $(0,t]$ given that the system entered the regenerative state S_i at $t=0$. The recursive relation for $R_i(t)$ are given by

$$R_i(t) = \sum_j Q_{i,j}(t) \circledR \left[\delta_j + R_j(t)\right] \quad (18)$$

Where j is any regenerative state to which the given regenerative state i transits and δ_i=1, if j is the regenerative state where the failed unit replaced by new ones, otherwise δ_i= 0. Taking$L.S.T.$ of relation(16) and solving for $R_0^{**}(s)$, we get the expected number of replacements per unit time as

$$R_0 = \lim_{s\to 0} sR_0^{**}(s) = \frac{N_7}{D_2} \quad (18)$$

Where $N_7 = \left(p_{46}\left(p_{14}+p_{02}p_{10}\right)\right)$ and D_2 is already specified.

IV. COST-BENEFIT ANALYSIS

Considering the various costs, profit incurred to the system model in steady state is given by:

Where $P = K_1A_0 - K_2B_0^r - K_3N_0 - K_4N_0^{pm} - K_5R_0$

- K_1 = Revenue per unit uptime of the system
- K_2 = Cost per unit time for which server is engaged in Preventive Maintenance, repair and replacement of the unit
- K_3 = Cost per unit visits by the server
- K_4 = Cost per unit visits by the server for maintenance
- K_5 = Cost per unit visits by the server for replacement

V. NUMERICAL RESULTS

The numerical results for MTSF, availability and profit function are obtained by considering exponential distributions for several random variables associated with repair time, maintenance time and replacement time of the unit as $g(t) = \theta e^{-\theta t}, p(t) = \rho e^{-\rho t}, f(t) = \beta e^{-\beta t}$.

By using the non-zero element p_{ij}, we obtain the following results:

$MTSF(T_0) = \frac{N_1}{D_1}$, Availability $(A_0) = \frac{N_2}{D_2}$,

Busy Period of server (B_0^r) $= \frac{N_3}{D_2}$,

Expected number of visits by server $(N_0) = \frac{N_4}{D_2}$,

Expected number of maintenance of the unit $(R_0) = \frac{N_5}{D_2}$,

Expected number of repair of the unit $(R_0) = \frac{N_6}{D_2}$,

Expected number of replacement of the unit $(R_0) = \frac{N_7}{D_2}$

Where $$N_1 = \frac{(b+a+\eta+\lambda)}{(b+\eta+\lambda)(a+\eta+\lambda)}$$

$$D_1 = \frac{(a+\eta+\lambda-ab)}{(b+\eta+\lambda)(a+\eta+\lambda)}$$

$$N_2 = \frac{(1+b)}{(b+\eta+\lambda)(a+\eta+\lambda)},$$

$$N_3 = \frac{\left(1+\lambda\left(\frac{\beta(1+\alpha_0)+\alpha_0}{(\theta+\alpha_0)\beta}\right)\right)(a+b+\eta+\lambda)}{(b+\eta+\lambda)(a+\eta+\lambda)},$$

$$N_4 = \frac{b(a+b+\eta+\lambda)}{(b+\eta+\lambda)(a+\eta+\lambda)},$$

$$N_5 = \frac{\frac{\theta\lambda}{(\theta+\alpha_0)}\left(\lambda(a+\eta+\lambda)+1\right)}{(b+\eta+\lambda)(a+\eta+\lambda)},$$

$$N_6 = \frac{\left(\lambda(a+\eta+\lambda)+\eta b\right)}{(b+\eta+\lambda)(a+\eta+\lambda)},$$

$$D_2 = b\left(1+\left(\frac{\lambda}{a}+\frac{2\eta}{\rho}+\lambda\left(\frac{\beta+\alpha_0}{(\theta+\alpha_0)\beta}\right)\right)\right)+(a+\eta+\lambda)\left(\left(1+\frac{\eta}{\rho}+\lambda\left(\frac{\beta+\alpha_0}{(\theta+\alpha_0)\beta}\right)\right)\right),$$

VI. ANALYSIS AND DISCUSSION

To observe the effect of the maintenance on the system performance and characterize the behavior of reliability measures MTSF, Availability and profit of the system, we make the tabular representation of the above numerical results for fixed values of parameters including $K_1 = 10000$, $K_2 = 1700$, $K_3 = 450$, $K_4 = 200$, with $a = 0.6$ and $b = 0.4$.

It can be seen in table 1 that MTSF is decline with the growth of constant rate of operation time (η). And, there is a further decline in its values when failure rate increases. as shown. Table 2 and 3 respectively revealed that availability and profit of the system model decrease with the increase of maximum rate of operation (η), constant rate of repair time (α_0), failure rate (λ). However, these reliability measures continuously increasing with the rise of repair rate (θ) and replacement rate (β). Also, the variation in the maintenance rate (ρ) of the unit improve the availability but decrease profit.

Table 1: MTSF vs. Operation Time

η	θ=2.1, α0=.13, β=.12, ρ=1.32, λ=.2, a=.6, b=.4	θ=2.1, α0=.23, β=.12 ρ=1.32, λ=.2, a=.6, b=.4	θ=2.1, α0=.13, β=.12, ρ=1.32, λ=.4, a=.6, b=.4	θ=2.1, α0=.13, β=.12, ρ=2.3, λ=.2, a=.6, b=.4
1	0.833	0.833	0.714	0.833
2	0.455	0.455	0.417	0.455
3	0.313	0.313	0.294	0.313
4	0.238	0.238	0.227	0.238
5	0.192	0.192	0.185	0.192
6	0.161	0.161	0.156	0.161
7	0.139	0.139	0.135	0.139
8	0.122	0.122	0.119	0.122
9	0.109	0.109	0.106	0.109
10	0.098	0.098	0.096	0.098

Table 2: Availability vs. Operation Time

η	θ=2.1, α0=.13, β=.12 ρ=1.32, λ=.2, a=.6, b=.4	θ=2.1, α0=.23 β=.12 ρ=1.32 λ=.2, a=.6, b=.4	θ=2.1, α0=.13 β=.12 ρ=1.32 λ=.4, a=.6, b=.4	θ=2.1, α0=.13 β=.22 ρ=1.32 λ=.2, a=.6, b=.4	θ=2.1, α0=.1 β=.12 ρ=2.3 λ=.2, a=.6, b=.4	θ=4.1, α0=.13 β=.12 ρ=1.32 λ=.2, a=.6, b=.4
1	0.467	0.453	0.422	0.477	0.568	0.487
2	0.341	0.334	0.318	0.346	0.453	0.352
3	0.270	0.265	0.256	0.273	0.378	0.276
4	0.223	0.220	0.214	0.226	0.324	0.228
5	0.191	0.189	0.184	0.192	0.284	0.194
6	0.167	0.165	0.161	0.168	0.253	0.169
7	0.148	0.146	0.144	0.149	0.228	0.150
8	0.133	0.132	0.130	0.134	0.207	0.134
9	0.121	0.120	0.118	0.121	0.190	0.122
10	0.111	0.110	0.108	0.111	0.175	0.112

Table 3: Profit vs. Operation Time

η	θ=2.1, α0=.13 , β=.12 ρ=1.32, λ=.2, a=.6, b=.4	θ=2.1, α0=.23 β=.12 ρ=1.32 λ=.2, a=.6, b=.4	θ=2.1, α0=.13 β=.12 ρ=1.32 λ=.4, a=.6, b=.4	θ=2.1, α0=.13 β=.22 ρ=1.32 λ=.2, a=.6, b=.4	θ=2.1, α0=.1 β=.12 ρ=2.3 λ=.2, a=.6, b=.4	θ=4.1, α0=.13 β=.12 ρ=1.32 λ=.2, a=.6, b=.4
1	3267	3244	3240	3304	2437	3348
2	2456	2432	2397	2485	1856	2523
3	1972	1950	1910	1995	1500	2027
4	1648	1628	1590	1668	1259	1696
5	1416	1398	1363	1433	1085	1457
6	1242	1226	1193	1256	953	1278
7	1106	1091	1061	1119	850	1138
8	996	983	955	1008	767	1026
9	907	894	869	918	699	934
10	832	820	796	842	641	857

VII. CONCLUSION

On the basis of above discussion it can be observed that maintenance rate of such a system plays no role for its MTSF but it increases the value of availability rapidly and decrease its profit. Thus performance of a single unit system with preventive maintenance and random arrival of server can be improved by either rising the repair rate of server or reducing the maintenance rate of unit. Furthermore replacement of the unit by new ones can also increase the profit and reliability of the system.

REFERENCES

[1] Al-Ali. Abdul Ameer, K. Murari, "*One unit reliability system subject to random shocks and preventive maintenance*", Microelectronics Reliability, 28 (3), 373–377, 1988.

[2] S. Chander, "*MTSF and profit evaluation of an electric transformer with inspection for on-line repair and replacement*, Journal of Indian Society for Statistics and Operations Research, XXVIII (1-4*)*, 33-43, 2007.

[3] M. S. Kadyan., Promila , "*Reliability and cost-benefit analysis of a single unit system with degradation and inspection at different stages of failure subject to weather conditions*", International Journal of Computer Applications, 55(6), 33-38, 2012.

[4] S.C. Malik, M.S. Barak, "*Reliability and economic analysis of a system operating under different weather conditions*", J Proc Natl Acad Sci, 79 (Pt II), 205–213, 2009.

[5] S.C. Malik, Gitanjali., "*Analysis of a parallel system with maximum repair time and single server who appears and disappears randomly*", International Journal of Mathematical Archive-3(11), 4700-470, 2012.

[6] T. Nakagawa, S. Osaki, "*Reliability analysis of a one-unit system with unrepairable spare units and its optimization applications*", Quarterly Operations Research, 27(1), 101-110, 1976.

[7] D. Pawar, D, S. C. Malik, "*Performance measure of a single-unit system subject to different failure modes with operation in abnormal weather*", International Journal of Engineering Science and Technology (IJEST), 3(5*)*, 4084-4089, 2011.

[8] Liu. Renbin, Liu. Zaiming, "*Reliability analysis of a one-unit system with finite vacations".* In the Proceedings of the MSIE 2011 International Conference on Management Science Industrial Engineering (MSIE), 248-252, 2011.

[9] S.K. Singh, "*Profit evaluation of a two-unit cold standby system with random appearance and disappearance time of service facility*", Microelectron. Reliab., Vol. 29, No. 1, pp.21-24, 1989.

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