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Particle Swarm Optimization Technique for Optimizing Conditional Value-at-Risk Based Portfolio

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***Abstract*- In existence of instability within the financial dealings, a reasonable harmony among risks and returns has to be managed by an investor to derive at an optimum standpoint. Although there is a predominant instability, the advantage lies in the correlation of the combination of financial instruments/assets in a financial portfolio within a specific market condition. Portfolio management targets the risk-reward accord in allocation of investments directed towards numerous assets for maximizing returns or minimizing risks within a stipulated investment period. This article delineates the particle swarm optimization algorithm, followed by optimized portfolio asset distribution within a changeable market condition. The suggested way is consolidated for optimization of the Conditional Value-at-Risk (*CVaR*) measurement within divergent market conditions established on numerous targets and restraints. Results are compared to the values obtained by the optimization of Value-at-Risk (*VaR*) measurement of the portfolios under consideration.**

***Keywords*- Portfolio Management, Risk-return paradigm, Value-at-Risk, Conditional Value-at-risk, Particle Swarm Optimization**

I. INTRODUCTION

In existence of instability in today's real work-a-day world's financial dealings, a reasonable harmony among risks and returns has to be managed by an investor to derive at an optimum standpoint. Although there is a predominant instability, the advantage lies in the correlation of the combination of financial instruments/assets in a financial portfolio in a stipulated market scenario. Of late, portfolio management is necessitated wanting for making decisions within the circumstances to invest involving a high-risk frame hence proving the present day's scenario risks and returns to be inevitably interlinked resulting in the importance in the decision making procedure in the investment opportunities. It directs the risk-reward accord for allocation of investments in numerous assets in turn maximizing returns or minimizing risks within a stipulated investment time frame. According to Markowitz, choosing an asset should not be done depending only on its characteristic features but also taking into account its co-movement with other assets [1]. Computation of risk as standard deviation of returns was done by Markowitz also showing diversification into different investment factors which in turn have limited or negative correlations in relation to their movements reducing overall risk [1]. This movement is measurable by a correlation coefficient varying between + 1 and -1 conferred to Markowitz [1].

There have been several general models to select portfolio which have come up over the years. These include the early mean variance models based on Markowitz's work in 1952 [1]. Of late, a host of stochastic optimization procedure established on the market scenario has assumed importance [2, 3, 4, 5, 6, 7]. No matter which model one resorts to, the underlying principle/notion lies in the minimization of some measures of market risks while simultaneously focusing on maximizing the portfolio return. It is noted that the risk metric is assumed as the function of the possible portfolio returns nearly in all models.

The most widely used method for assessing downside risk within a portfolio is the Value-at-Risk (*VaR*) which is characterized as the p^{th} percentile of portfolio return at the edge of the planning horizon. Incidentally, for low values of p (as low as 1, 5 or 10), it identifies the “worst case” outcome of portfolio returns. Stambaugh (1996) envisages *VaR* to be (1) a terminology for risk, (2) giving space to efficient and coherent risk management, (3) providing an enterprise-wide technique for market regulation, and (4) acting as a tool for risk assessment [8].

A score of literature exists regarding the varied techniques for the computation/calculation of a portfolio's *VaR*. One of the interesting perspectives of *VaR* derivation is ascertaining how it can be applied for allocation of portfolios in multifinancial instruments situation. If business organizations perform based on *VaR*, then it stands to be a vital issue in designing the strategy for investment selection. Furthermore, if organizations take decisions in a *VaR* context, then the implications of the organization's risks are to be taken into cognizance. Since *VaR* is rather discrete in nature and is difficult to incorporate in traditional stochastic models, not much work has been reported in the literature with regards to its optimization attempts. Rockafellar and Uryasev (2002) proposed a scenario-based model for portfolio optimization [9]. They adopted the Conditional Value-at-Risk (*CVaR*) for this purpose. *CVaR* is described to be the amount of expected loss outrunning *VaR*. Their illustration curtails *CVaR* in course of calculating *VaR*. It was observed that the minimum-*CVaR* is equivalent to the minimum-*VaR* concerning of regularly apportioned portfolio returns.

Thus for measuring the financial worth of an asset or of an asset's portfolio within the market which gets decreased during a stipulated period of time (usually considered during 1 day or 10 days) subservient to typical market conditions, Value-at-Risk (*VaR*) is considered as an effective tool. It is also highly valued for being incorporated within industry regulations regardless of suffering from the instability along with difficulty to work using numerical values if there is normal distribution of losses because loss distribution often tends to exhibit "fat tails" or empirical discreteness.

Value-at-Risk (*VaR*) at a confidence level is thus treated as the maximum amount of loss not outstripping at a stated probability, over a stipulated time period. *VaR* has always been determined by three parameters, viz., (i) the stipulated time period (typically 1 day, 10 days, or 1 year) which is to be analyzed as the time over which any organization should hold its portfolio, or to the time required for liquidating its assets, (ii) the level of confidence (common values are 99% and 95%), which is the approximated value for the interval where the *VaR* would not likely exceed the unit of *VaR* in currency, and (iii) the maximum probable loss structure.

Unlike Value–at-risk, Conditional Value-at-Risk (*CVaR*) stands as the risk measuring technique in case of risk having significant advantages, for deriving distribution of losses in finance involving discreetness [9]. Because of the customariness of different proposed structures found on varied scenarios and finite sampling, the utilization of such distributions have become an important property in the economic markets in turn.

CVaR can be identified to be the mean value of *VaR* along with $CVaR^+$ (the values themselves be contingent on the decision x along with the weights), where none of the values for *VaR* and *CVaR*+ stand to be coherent. The specific method for computing *CVaR* in relation to probability of *VaR* value gives the worth of weights, in existence of the other.

Computational advantages of *CVaR* over *VaR* prove to be the leading impetus in the *CVaR* methodology development procedure, despite of substantial efforts for finding out the effective algorithms for optimization procedure of *VaR* in high-dimensional environments which are still unavailable. *CVaR* proved as a new coherent risk measuring structure having distinct advantages when been compared to *VaR*, quantifying risks beyond *VaR*, consistent at divergent positions of confidence α (smooth relating to α) and also being a static statistical estimate with integral characteristics [9]. *CVaR* has thus been entrenched as an excellent tool in the risk management procedure and optimization of portfolio accompanied by linear programming having huge dimensions in the company of substantial numerical implementations. At various time periods with divergent positions of confidence, distributions are also shaped for multiple risk constraints along with the previously mentioned tasks, which in turn stand as swift algorithms for online usage. Rockafellar and Uryasev (2002) have considered *CVaR* methodology as a consistent one having mean-variance method considered in turn as minimal portfolio (with return constraint), which can also be accepted as a variance minimal in terms of normal loss distribution [9].

Within this research work, an algorithm exercising particle swarm optimization has been used for evolving optimized portfolio asset allocations within instable financial dealings. The suggested way is to optimize the Conditional Value-at-Risk (*CVaR*) measurement in divergent market scenarios in terms of numerous targets and restraints [9]. Other than implementing the general definition of *CVaR* and its minimization formulae associated, the writers have concentrated here into assessing the entire various distributions enhancing the usefulness and worth of *CVaR* in case furnishing the elementary way of calculating *CVaR* directly. The results obtained have been put into comparison with those ascertained from the optimization of Value-at-Risk (*VaR*) measurement procedure of the portfolios on selected financial tool along with a real life data set of TATA Steel from mid-August and early September, 2015, enabling a distributional assumption for employing the particular type of financial assets for developing a much more generalized framework.

The authors have planned the article into the following different sections providing an overview of the conventional concept of Conditional Value-at-Risk in section II. Section III then demonstrates the mathematical assertion of *VaR* along with *CVaR* measurement. A discussion of the particle swarm optimization process along with the algorithm associated is provided in section IV. The findings are summarized in section V. Conclusions added to future directions of research are drawn out finally in section VI.

II. CONVENTIONAL CONCEPT OF CONDITIONAL VALUE-AT-RISK

Introduction of return risk management framework by Markowitz (1952) has come a long way in procedure of portfolio optimization [1]. Of late, the utilization of alternative coherent technique is been done for the reduction of probable amount of losses within a portfolio. This can be worked out by assessing of the specific loss that will be exceeding the value at risk. The outcome risk measurement is termed to be the Conditional Value-at-Risk (*CVaR*) [9]. Appreciation goes to the evolving fields of data intelligent management and archival techniques in industrial portfolio management. Simulation by fundamental requirements in turn has developed within portfolio optimization procedure by (i) risks, constraints and adequate modeling of utility functions and (ii) efficient handling of huge number of scenarios and instruments. In mathematical understandings, derivation of *CVaR* is done by considering the mean value at the intervals of the value-at-risks and the amount of loss outstripping the value-at-risks. *CVaR* being compared to *VaR*, it not only traces several different loss distributions but can even also be easily expressed in minimization formula.

Measures of risk play an indispensible act especially in coping with losses which might have been incurring in finance under the shed of uncertain conditions. Loss, being derived as a function z = $f(x, y)$ within a decision heading $x \in X$ representing the values of numerous variable viz. interest rates or weather data in terms of the future values. If y is assumed to be random with known probability distribution z, then it turns out to be a variable random in nature having its dependent distribution on the choice of x. If any optimization problem shows the involvement of z in spite of the choice of x, then it can be accounted not just as expectations but even as "riskless" of x.

Percentile measures of loss or reward can be done by $f(x,y)$, which is taken to be the loss function depending upon the decision vector $x = (x_1,....x_n)$ and the random vector $y = (y_1,....y_m)$, then *VaR* is calculated as α-percentile, representing the loss distribution which is treated as the minimal value where the probability which loss exceeds or is equivalent to the value which is exceeding or equaling to α. In such case, $CVaR^+$ which is also known to be the "upper *CVaR*" is the loss expected, which strictly exceeds *VaR* (in turn known as Mean Excess Loss) and Expected Shortfall. $CVaR^-$ in turn known as "lower *CVaR*" is the loss expected weekly exceeding *VaR*, which is the loss expected equal to or exceeding *VaR*. It is also popular as Tail *VaR*.

Thus, *CVaR* is the mean value of *VaR* and $CVaR^+$ [9]. It can be derived from the following formula:

$$CVaR = \lambda VaR + (1 - \lambda)\, CVaR^+, \qquad 0 \le \lambda \le 1 \quad (1)$$

Where, λ is the Lagrange multiplier.

Considering *CVaR* is convex as shown in "Fig.1", *VaR*, $CVaR^+$, $CVaR^-$ can also be non-convex. This shows credible inequalities as:

$VaR \le CVaR^- \le CVaR \le CVaR^+$ (Rockafellar and Uryasev, 2002) [9].

The relationships between *VaR*, *CVaR*, $CVaR^-$ and $CVaR^+$ (Rockafellar and Uryasev, 2002) are shown in "Fig.2" [9].

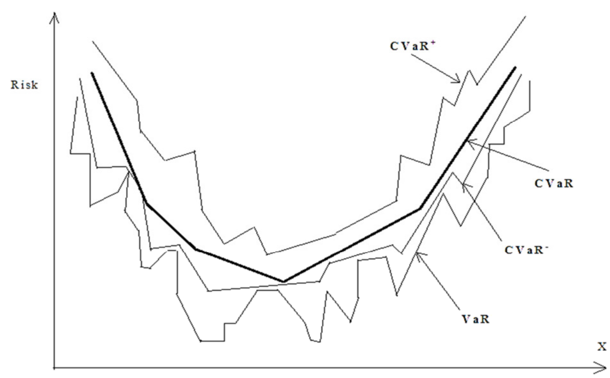

Fig.1. CVaR: Convex Function (Rockafellar and Uryasev, 2002) [9].

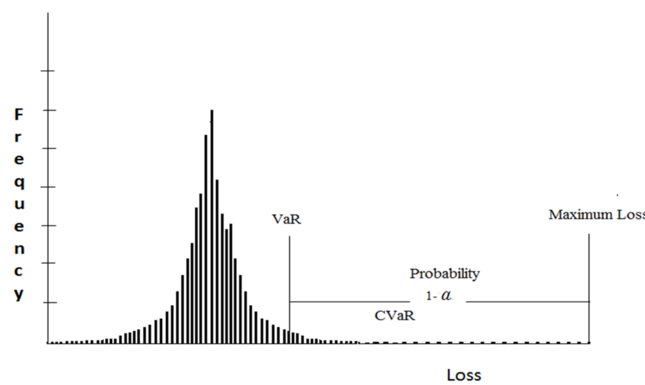

Fig.2. The relationships between *VaR*, *CVaR*, $CVaR^-$and $CVaR^+$ (Rockafellar and Uryasev, 2002) [9].

III. MATHEMATICAL FORMULATION

VaR is treated as an important approach for the disclosure of a stipulated financial portfolio within the varied risk situations, instinctive in financial structures, which in turn is also having paramount significance within a portfolio optimization purpose.
Considering a portfolio *P*, levelheaded by *k* assets, $S = \{S_1, S_{2...}S_k\}$, and $W = \{W_1, W_{2...}W_k\}$ considered as the relative weights or portions of the assets within the stipulated portfolio, the price can be estimated as:

$$P(t) = \sum_{i=1}^{k} S_i(t) W_i \qquad (2)$$

Within which $S_i(t)$ and W_i stand to be the values and importance levels of the portfolio within a stipulated time period t, respectively.
The *VaR* of the stated portfolio *P*, which is the maximum amount of loss expected within a stipulated holding period at a stated level of confidence (α), then can be considered to be the smallest number l such that the probability of the loss termed as L exceeds l is not larger than (1 - α), i.e.

$$VaR_\alpha = \inf\{l \in R : P(L > l) \leq 1 - \alpha\} = \inf\{l \in R : F_L(l) \geq \alpha\} \qquad (3)$$

All the techniques and models depend on presumptions of their own. However, the common presumption that stands to be the best estimator for future changes in market conditions is the historical trace of available market data. Few of the acclaimed models for estimating *VaR* include:

1. Variance-Covariance (VCV) model – It is helpful for the assumption of the risk factor returns which is to be normally (jointly) distributed in every cases, and at the same time the portfolio return which in turn is also normally distributed. It is also helpful in assumption of the modification in the portfolio's worth which is dependent directly on every risk factor returns. In 1990, this model got popularized. The assumption of portfolio's return to be normally distributed shows the composition of assets within a portfolio. The changes/deltas being linear state, changes within the value of portfolio which in turn is dependent linearly on each and every changes in the values of the assets. This implies portfolio's return is also dependent linearly on every asset returns and the asset's return is moreover jointly normally distributed. With further assumption that the risk factor connected to a stated financial portfolio is the portfolio's worth itself, the 95% level of confidence *VaR* for *N* assets within a holding period, is given by

$$VaR = -V_p(\mu_p - 1.645\sigma_p) \qquad (4)$$

In which, the mean μ_p is given by

$$\mu_p = \sum_{i=1}^{N} \varpi_i \mu_i \qquad (5)$$

The standard deviation σ_p is given by

$$\sigma_p = \sqrt{\Omega^T \Sigma \Omega} \qquad (6a)$$

$$\Omega = \begin{bmatrix} \varpi_1 \\ \varpi_2 \\ \varpi_3 \\ . \\ . \\ \varpi_N \end{bmatrix} \qquad (6b)$$

$$\Omega^T = [\varpi_1 \quad \varpi_2 \quad \varpi_3 \quad . \quad . \quad \varpi_N] \qquad (6c)$$

Where, i refer to the return on asset, i and p refer to the portfolio's return for standard deviation (σ_p) and mean (μ_p). V_p is taken to be the portfolio's worth at the beginning (in currency units). ϖ_i is hence expressed to be the ratio of V_i to V_p.

If compact and maintainable data set has been purchased from third parties, VCV model stands to be beneficial in context of their usage and also in the speed of calculation by the usage of optimized linear algebraic libraries. The main drawbacks of this model lie in the presumption that the portfolios generally comprise assets for which delta is linear and that the market price returns/asset returns are distributed normally.

2. Historical simulation (*HistSim*) model – Emerging as the industry standard for calculation of *VaR,* in context to the presumption that the asset's return in future always will be providing an equal amount of distribution as happened in the past. Hence, *HistSim* is treated as the simplest and transparent method for calculating the *VaR*. This model for computation of a percentile (*VaR*) associated within the prevailing set of portfolio across a set of historical trace for yielding modifications in the portfolio value. Its simplicity of implementation stands to be its most important benefit along with not assuming a distribution normal in nature of asset returns like the VCV model. Its intensive calculation computationally along with the necessity for a larger market scenario fall under its main drawbacks.
In *HistSim*, *VaR* is evaluated as:

$$VaR = 2.33 M \sigma_p \sqrt{10} \qquad (7)$$

Where, M is considered as the market worth of a portfolio and σ_p is treated as the historical instability of that portfolio. The constant 2.33 stands for the number of σ_p which is required for a level of certainty of 99% and the constant √10 refers to the number of days in the holding period.

Basically, calculation of VaR is done in the *HistSim* method in two simple steps. Firstly, construction of a series of pseudo historical portfolio returns is calculated by the usage of portfolio weights along with historical asset returns. Secondly, the calculation of *VaR* and the prevailing asset returns quantile of the pseudo historical portfolio returns are carried out.

3. Monte Carlo simulation – This model basically undergoes the random simulation of future asset returns. Usage of this simulation is done generally for calculating *VaR* for portfolios which are containing the securities with non-linear returns and in which the requirement of computational effort is non-trivial. Conceptually, simplicity of this method stands to be its added advantage, still it is computationally more intensive than both the VCV and *HistSim* models. The generic Monte Carlo *VaR* calculation incorporates the following steps:

    1. Predefining *N*, denoting the number of iterations which is to be performed.
    2. For every iteration in *N*,
        - Generating a random scenario of market which moves by the usage of some existing model present in the market.
        - Revaluing a portfolio under the simulated market volatility scenario.
    3. Computing a portfolio's profit or loss (PnL) in case of the simulated scenario and for doing so, subtracting the current market worth of that portfolio from its market value which has already been estimated in the last previous steps.
    4. Sorting the result *PnLs* required for obtaining the simulated Profit and Loss (*PnL*) distribution within the stated portfolio.
    5. Finally, calculation of *VaR* at a stated level of confidence by using the function of percentile.

The features of *CVaR* represent the risks which are simple and convenient in nature hence measuring the downside risks and are applicable to non-symmetric distribution of losses. Stable statistical estimates of *CVaR* appear to be its integral characteristics in comparison to *VaR* which can get influenced by any scenario. *CVaR* yields values in a continuous process in respect to the confidence level α, steady at divergent phases of confidence in comparison with *VaR* (*VaR*, $CVaR^-$, $CVaR^+$ may not be continuous to α). *CVaR* portfolios coexist within normal distribution of loss in optimal variance to the level of consistency in mean variance approach. *CVaR* is variedly acceptable due to its easy control and optimization process for non-normal distributions, even shaping of loss distribution is being done using *CVaR* constraints for the first online procedures.

IV. PARTICLE SWARM OPTIMIZATION

The occurrence of evolutionary estimation has been inspiring new resources for optimizating in different problem solving procedures variably within the area of portfolio management. Evolution of algorithms, like Genetic Algorithm (GA), Ant Colony Optimization (ACO), Simulated Annealing (SA) along with Particle Swarm Optimization (PSO), all of which tend to assess the global solution of a stated problem [10,11, 12, 13].These algorithms have come to usage as effective tools for evaluation of numerous points in search space simultaneously.

Based on the simulation of simplified social models viz. bird flocking, fish schooling, along with the swarming theory, PSO stands to be evolutionary computation mastery in context to individual enhancement along with population cooperation and competition. The concept desires primitive mathematical computations, which is not at all computationally expensive in consideration to memory requirements in terms of irrespective period of time. Optimization of fitness function is evaluated for every particle. The desired value for the comparison purpose of the particle's fitness worth with particle's *pbest* (personal best) is found out and if prevailing worth is better than *pbest*, *pbest* 's worth is set which in turn is equal to the prevailing worth and is equal to the current position in a *K*-dimensional space. Comparing fitness worth of the particle with the particle's fitness values obtained so far falls into the next procedure. If the prevailing worth stands to be better than the *gbest* (global best), then the *gbest* value is reset to the prevailing particle's worth. Repetition of this mechanism gets continued till the criterion for stopping is met.

In the PSO mechanism, the representation of each potential particle representing a particle with a position vector x, refers to the phase weighting factor b and a moving velocity v, respectively. For *K* dimensional optimization, the location along with the velocity of the i^{th} particle are represented as $\mathbf{b}_i = (b_{i,1}, b_{i,2}, \ldots, b_{i,K})$ and $\mathbf{v}_i = (v_{i,1}, v_{i,2}, \ldots, v_{i,K})$, respectively. Each particle having its individual top position $b_i^P = (b_{i,1}, b_{i,2}, \cdots, b_{i,K})$ which corresponds to the individual best objective value which is obtained till the time *t*, is referred to as *pbest.* The best particle globally (*gbest*) is expressed by $\mathbf{b}^G = \left(b_{g,1}, b_{g,2}, \cdots, b_{g,K}\right)$, which is represented as the top particle so far at time *t* within the entire swarm. The recent velocity $\mathbf{v}_i(t+1)$ for particle *i* is reconditioned by

$$v_i(t+1) = wv_i(t) + c_1[b_i^P(t)] + c_2\left[b^G(t) - b_i\ (t)\right] \quad (8)$$

where, w is supposed to be the *inertia weight*, $v_i(t)$ is considered to be the old velocity for the particle i at time period t. Seemingly from the above stated equation, the newly obtained velocity in relation to the velocity which is found in the previous course which in turn is denoted by w (weight) and is also affiliated to the place of the particle with that of the global best one by acceleration constants c_1 and c_2 . Accelerations c_1 and c_2 , being the constants adjust to the value of tension in PSO system. High values fetching abrupt movement indicating the target areas in context to the lower values allows the particles to move far from the target areas before being tugged back. The acceleration constants c_1 and c_2 are therefore been stated to be the cognitive and social rates for representing the weighting factor of the acceleration terms, pulling each particle towards the best positions in respect to personal and global spectrum. Particle swarm optimization is thus a population-based stochastic optimization procedure which has been originated from the social behavior of bird flocking or fish schooling. Thus PSO is considered as every solution like a “bird” (particle) in the search area of food (the best solution). All particles having fitness values are evaluated by the fitness function, having velocities that direct the “flying” (or evaluation) of the particles which in turn begins with a set of random particles (solutions). PSO searches for the optimal solution by updating generations in each iteration. All particles are updated by the “best” values already stated as the *pbest* or personal best stating the best solution or fitness which a particle has obtained till that moment in turn the other is the *gbest* or global best stating the best value obtained by any particle in the population. The best value achieved within the topological neighbour or within the segment of a population is a local best known to be the *pbest*. Finding the best values, velocity and position for the particle are updated using "(9)" [14].

$$V_{t+1}iX_{(t+1)i}=wV_{ti}+c_1\mathrm{rand}_1()(pbest_i-X_{ti})+c_2\mathrm{rand}_2()(gbes_{ti}-X_{ti})=X_{ti}+V_{(t+1)I} \quad (9)$$

Where, i is once again treated as the index of each particle, t is the current iteration number, $\mathrm{rand}_1()$ and $\mathrm{rand}_2()$ are random numbers within 0 and 1. $pbest_i$ is the best experience in previous course of the i^{th} particle while $gbest_i$ stands to be the best particle within the population as a whole. Constants c_1 and c_2 are considered as the weightage factors of the stochastic acceleration terms, which in turn exherts every particle toward the $pbest_i$ and $gbest_i$, w being the inertia weight controlling the exploration characteristics of the algorithm. If $c_1 > c_2$, the particle tends to reach $pbest_i$, the best position identified by the particle, rather than converge to $gbest_i$ found by the population and vice versa.

The procedure of PSO starts with a population group generated randomly, both in turn having fitness values for evaluating the group of population. Population can be updated and searched for the optimum with random techniques in both the procedures without any success been guaranteed.

However, unlike the genetic algorithm, PSO does not include genetic operators such as crossover and mutation. With the internal velocity, particles start updating themselves having memory, which stands to be of having most importance to the algorithm. PSO proves to be very differently significant, when terms of comparison arise in genetic algorithms (GAs). Chromosomes in turn share knowledge among each other for moving whole population like one group enhancing an optimal domain where only *gbest* (or *lbest*) gives out the information to others. Hence it proves to be mechanism for sharing information on one way basis in turn proving the evaluation just looking for the best solution, along with every particle tending for the best result quickly even in the local version in most of the cases.

V. PROPOSED METHODOLOGY

The suggested mechanism is targeted for optimization of the Conditional Value-at-Risk (*CVaR*) measures within a portfolio comprising numerous financial instruments at divergent market scenarios based on numerous goals and constraints. Application of PSO for *CVaR* optimization is demonstrated in respect to the minimization of the risks within the portfolios under consideration, thereby minimizing the portfolio losses incurred. The flow diagram of the suggested mechanism is shown in "Fig.3".

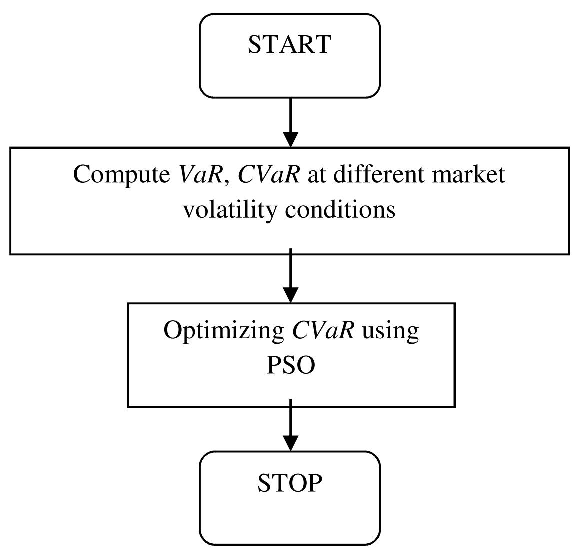

Fig.3. Flow diagram depicting the proposed methodology

The procedure of portfolio asset allocation optimization is demonstrated on a collection of 20 portfolios with several asset variations. Here in this process of optimization, the particle swarm optimization algorithm has been run with two different numbers of generations viz., 500 and 1000 with the constants already been specified in Table I.

TABLE I. PARTICLE SWARM OPTIMIZATION PARAMETERS EMPLOYED

Sl. No.	PSO Parameter	Values used
1.	Number of Generations	(500, 1000)
2.	Inertia weight	0.8
3.	Acceleration Coeffient (ϕ_1)	1.5
4.	Acceleration Coeffient (ϕ_2)	1.5

The optimization of the portfolio asset allotment is achieved with particle swarm optimization [15]. In terms of faithfully allocating the assets within a given level of confidence, it minimizes the Conditional Value-at-Risk (*CVaR*) for the portfolio using "(10)" as the fitness function.

$$CVar = \frac{e^{-(\frac{VaR^2}{2})}}{a\sqrt{2\pi}} \qquad (10)$$

Where, *a*=0.01 considering a confidence level of 99% and *VaR* are the Value-at-Risk measures of the portfolios under consideration.

Table II lists the different archived average optimized portfolios over two different numbers of generations along with their costs for a confidence level of 99%.

In addition, as a comparative study, the Historical Simulation (*HistSim*) model has been used for calculating *VaR* of the portfolios under consideration. The particle swarm optimization algorithm is then utilized for obtaining optimum *VaR* measures of the portfolios using "(7)" as the fitness function. The optimized *VaR* values obtained using the particle swarm optimization algorithm is also delineated in Table II for the necessity of comparison.

TABLE II. COMPARATIVE RESULTS OF OPTIMIZED PORTFOLIOS WITH THEIR COSTS, *CVAR*S AND *VAR*S AT A CONFIDENCE LEVEL OF 99%

Portfolio No.	Portfolio Cost (in currency units)	*CVaR*	*VaR*
1	26758.782629	0.190600	0.224732
2	29942.989518	0.185829	0.226188
3	34868.339288	0.183854	0.224514
4	36591.431339	0.184595	0.224596
5	35996.607129	0.185289	0.224314
6	32243.667175	0.181077	0.224495
7	30576.016315	0.182792	0.225213
8	33300.584075	0.186494	0.225019
9	30446.646374	0.181387	0.225806
10	29874.663429	0.187987	0.225377
11	36049.580870	0.182420	0.224926
12	33303.676941	0.184245	0.225910
13	38518.502646	0.179264	0.224766
14	47675.001370	0.177929	0.224173
15	26358.703336	0.193556	0.226707
16	30098.635906	0.189505	0.225343
17	35391.282927	0.184244	0.224860
18	24874.327332	0.182948	0.224615
19	33692.352544	0.184193	0.225842
20	36283.390566	0.180552	0.224320

From Table II, it is evident that the *CVaR* measures provide a more realistic impression for allocating the financial instruments since for all the 20 portfolios, the *CVaR* measures reflect minimized market risks as compared to their *VaR* measures.

It can be observed from Table II and Table III that PSO delineates a faithful selection/allocation strategy of assets within the financial portfolio which can be derived by the usage of an optimization procedure in particular iteration and generated a solution which is local optimum. Moreover, PSO further explores to find much optimized solution using *CVaR* techniques than the one generated by PSO by usage of *VaR* techniques. The proposed approach aims at minimizing the *CVaR* measures of the portfolios under consideration by the usage of particle swarm optimization technique. The optimized portfolios are seen to outperform the corresponding *VaR* based optimized portfolios with regards to minimization of market risks. As far as our knowledge is concerned, no such attempts have been reported in the literature so far. Hence, this initiative is a maiden venture in this direction.

VI. CONCLUSION

Within the domains of economics and finance, portfolio management has proved having the supreme importance as a systematic discipline given at the strategies for diversification in investment. This research work targets to evolve a selection and allocation strategy of portfolios within an unstable market scenario by means of the optimization of the Conditional Value-at-Risk (*CVaR*) measures of the portfolios under consideration. A particle swarm optimization mechanism is adopted on historical portfolio data and a real life data set of TATA Steel for this purpose which in turn is delineated in Table III. Faithful selection of results is exhibited on different portfolios with several asset combinations.

TABLE III. COMPARATIVE RESULTS OF OPTIMIZED PORTFOLIOS OF TATA STEEL LIMITED WITH THEIR COSTS, *CVAR*S AND *VAR*S AT A CONFIDENCE LEVEL OF 99%

Date	Symbol	Open Price	High Price	Low Price	Last Traded Price	Closing Price	Total Traded Quantity	Turnover (in Lakhs)	Difference Of Closing Prices of Consecutive Days	*CVaR*	*VaR*
2/9/15	TATA STEEL	218.9	225.9	210.3	220.9	219.5	8909770	19476.16	3.200000	0.070347	0.199387
1/9/15	TATA STEEL	222	226.25	213.05	216.4	216.3	6475249	14253.2	-9.100000	0.061473	0.200462
31/8/15	TATA STEEL	225.3	228.7	223.65	225.3	225.4	5782337	13047.13	-3.550000	0.053436	0.205600
30/8/15	TATA STEEL	232.6	234.6	223.2	229.5	228.95	9387248	21607.97	2.550000	0.061694	0.205542
29/8/15	TATA STEEL	220	229.8	215.1	227.2	226.4	14227129	31569.67	10.750000	0.054159	0.204653
28/8/15	TATA STEEL	213	219.7	208.95	214.8	215.65	8336683	17819.13	2.200000	0.077069	0.194805
27/8/15	TATA STEEL	210	217.9	200.1	215	213.45	14206932	29799.94	7.350000	0.054156	0.203642
26/8/15	TATA STEEL	229.3	229.3	202.65	204.45	206.1	11711489	25572.86	-31.150000	0.068291	0.201481
25/8/15	TATA STEEL	238.5	238.5	231.5	236.3	237.25	5247042	12325.35	-4.300000	0.060858	0.203103
23/8/15	TATA STEEL	247.7	248.4	240.5	242	241.55	4626334	11255.37	-8.250000	0.068447	0.200799
22/8/15	TATA STEEL	250.1	253.35	246.75	249.55	249.8	4304085	10772.08	-2.550000	0.052530	0.205786
21/8/15	TATA STEEL	248.1	254.9	246.6	252.4	252.35	9805064	24641.63	5.600000	0.061528	0.200201
20/8/15	TATA STEEL	238.5	248.5	234.2	247.5	246.75	8524452	20665.62	9.150000	0.073113	0.200397
19/8/15	TATA STEEL	234.25	239.2	229	237.5	237.6	9065329	21282.27	4.000000	0.060508	0.200104
18/8/15	TATA STEEL	253.35	253.9	232.3	234	233.6	13764560	32979.36	-15.450000	0.050674	0.201994
17/8/15	TATA STEEL	254.4	257.9	246.1	249.45	249.05	15172684	38227.01	2.250000	0.066088	0.199147
16/8/15	TATA STEEL	259.95	260	245.7	247.3	246.8	8827248	22200.72	-14.350000	0.067079	0.201798
15/8/15	TATA STEEL	262.9	265	260	260.25	261.15	3401136	8929.59	-0.900000	0.061850	0.201672
14/8/15	TATA STEEL	261	264.55	260.15	262.05	262.05	3791638	9948.04	1.050000	0.069911	0.199146
13/8/15	TATA STEEL	262.35	265.4	256.25	260.25	261	6376740	16691.22	-1.300000	0.051213	0.206873
12/8/15	TATA STEEL	260	268.5	259.95	262.3	262.3	7979075	21145.9	6.000000	0.067111	0.199241
11/8/15	TATA STEEL	249.7	259.75	246.05	258.8	256.3	9723318	24527.2	-	-	-

CVaR being into a new risk measuring procedure provides significant advantages when been compared to *VaR* which in turn can quantify risks beyond the level of *VaR*. Hence it is known as a coherent risk measurement procedure which is consistent in different confidence levels. The writers have attempted evolving a strategy for selection and allocation of portfolios under different financial market conditions. For the stated purpose Particle Swarm Optimization (PSO) procedure is adopted under the above two mentioned conditions .Thus the proposed mechanism is targeted at minimizing the *CVaR* measures of the portfolio under consideration.

Methods however remain to be investigated to incorporate the aspect of return maximization in the portfolio allocation scenario through multi-objective optimization techniques. The authors are currently engaged in this direction.

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