

Single Sampling Plan for Variable Indexed by AQL and AOQL with Known Coefficient of Variation

J.R. Singh¹, A. Sanvalia^{2*}

¹ School of Studies in Statistics, Vikram University, Ujjain, India

^{2*} School of Studies in Statistics, Vikram University, Ujjain, India

*Corresponding Author: arvindsanvalia@gmail.com, Tel.: 0-77470-91864

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Abstract— In this paper we have studied the effect of coefficient of variation (CV) on single sampling plan for variables indexed by acceptable quality level (AQL) and average outgoing quality limit (AOQL) based on the assumption of normality and independence are affected when the characteristic of an item possesses a normal distribution. Procedures and tables are given for the selection of single sampling plans for variables for given AQL and AOQL, whenever rejected lots are 100% inspected for replacement of nonconforming units. The operating characteristic (OC) function is described for different values of coefficient of variation. It is clear that the value of OC function with known CV shows higher values for the lot of bad quality. The values of n and k are also calculated with known coefficient of variation.

Keywords— CV, sampling plan, AQL, AOQL, OC

I. INTRODUCTION

The goal of acceptance sampling is to decide if the lot is likely to be acceptable, not to estimate the quality of the lot. Acceptance sampling is used in quality control practice when the cost of 100% inspection is very high, 100% inspection takes too long, and/or testing is destructive. Acceptance sampling plan is an assessment procedure applied in statistical quality control; it is a technique of measuring random samples of populations called “lots” of products against predetermined standard. Acceptance sampling is a part of operations managing or of accounting auditing and services quality supervision. It is important for industrial, but also for business purpose helping decision-making procedure for the purpose of quality management. Nowadays, as more companies start to apply quality programs, such as Total Quality Management (TQM) approach, they work directly with suppliers to ensure high levels of quality and the need for acceptance sampling plans is decreasing. The aim is that no defective items should be entered into the production process, passed from a producer to a consumer, which could be an exterior or an interior customer. In actuality many firms must check their materials inputs. Designing an acceptance sampling plan is making a decision about quality and risk in acceptance sampling plan two levels of quality are considered: primary, acceptable quality level, and, secondary, the unacceptable quality level. The first is the quality level desired by the consumer and is called the limit quality or the acceptable quality level (AQL). The producer’s risk is the risk that the sampling plan will fail to confirm an acceptable lot’s quality AQL and, thus,

reject it. This kind of risk is also called a Type - I Error of the plan. A lot-to-lot rectification assessment scheme for a series of lots calls for 100% inspection of rejected lots under the application of a sampling plan. If it is preferable to use a single sampling plan for variables under a rectification inspection scheme, the index for the choice of the sampling plan will be the average outgoing quality limit (AOQL), which is the unacceptable average quality the consumer will receive in the long run, regardless of the incoming quality. Rejected lots are often a nuisance to the producers because they result in extra work and extra cost. If too many lots are rejected the status of the producer or supplier may be damaged. From the producer’s point of view, it is preferable to fix an acceptable quality level (AQL) by designing a sampling plan such that, if the incoming product quality is maintained at AQL most of the lots, for example 95%, will be accepted during the sampling inspection step. Thus, designing sampling inspection plans indexed by AQL and AOQL satisfies both the producer and consumer whenever rectifying inspection is necessary. Soundarajan (1981) developed procedures and tables for the selection of single sampling plan for attribute for given AQL and AOQL. Several authors have proposed predictors for estimating the rate of non-conformances in lots subjected to acceptance sampling (Hahn, 1986; Zaslavsky, 1988; Brush, et al. 1990; Martz & Zimmer, 1990, Govindaraju 1990, Govindaraju and Soundararajan 1986, Sankle and Singh, 2012a, Sankle and Singh 2012b).

In this paper we have studied the effect of coefficient of variation (CV) on single sampling plan for variables indexed

by AQL and AOQL based on the assumption of normality and independence are affected when the characteristic of an item possesses a normal distribution. Procedures and tables are given for the selection of single sampling plans for variables for given AQL and AOQL, whenever rejected lots are 100% inspected for replacement of nonconforming units. The operating characteristic (OC) function is described for different values of coefficient of variation. It is clear that the value of OC function with known CV shows higher values for the lot of bad quality. The values of n and k are also calculated with known coefficient of variation.

II. VARIABLE SAMPLING PLAN FOR AOQL AND OC FUNCTION WITH KNOWN CV

A variable sampling plan is generally used whenever the characteristics of interest is measurable on a continuous scale and is normally distributed with mean μ and standard deviation σ . A complete discussion on the application and operation of variables sampling plans can be seen in Wadsworth, Stephens and Godfrey (1986). In referencing a single sampling variable plan when σ and CV is known, the following symbols are used:

L: Lower specification limit;

U: Upper specification limit;

n_σ : Sample size;

k_σ : Acceptance parameter; and

\bar{x} : Sample mean

$$F(y) = \int_{-\infty}^y \left(\frac{1}{\sqrt{2\pi}}\right) \exp\left(-\frac{1}{2}z^2\right) dz. \quad (2.1)$$

Where $z \sim N(0, 1)$. The acceptance criterion for the single sampling plan is: For the upper specification limit, accept the lot if,

$$\bar{x} + k_\sigma \sigma \leq U, \quad (2.2)$$

and, for the lower specification limit L,

accept the lot if,

$$\bar{x} + k_\sigma \sigma \geq L. \quad (2.3)$$

The fraction nonconforming in a given lot will be

$$p = F(-v), \quad (2.4)$$

with

$$v = \frac{U - \mu}{\sigma}. \quad (2.5)$$

Where v is the p percent point of the standard normal distribution. If p is the proportion defective in the lot, then

$$U = \mu + v\sigma, \quad (2.6)$$

and its probability of acceptance under with known CV will be

$$P_a(p) = F(y), \quad (2.7)$$

with

$$w = (v - k_\sigma) \sqrt{n_\sigma + v^2}. \quad (2.8)$$

If the quality of the accepted lot is p and all nonconforming units found in the rejected lots are replaced by conforming units in a rectification inspection scheme, the AOQ can be approximated as

$$AOQ = pP_a(p). \quad (2.9)$$

If pm is the proportion non-conforming at which AOQ is maximum, then

$$AOQL = p_m P_a(p_m). \quad (2.10)$$

If $AQL(p_1)$ is prescribed, then the corresponding value of v_{AQL} or v_1 will be fixed, and if $P_a(p_1)$ is fixed at 95%, then, $w_{AQL} = w_1 = 1.645$. Hence, we have

$$1.645 = (v - k_\sigma) \sqrt{n_\sigma + v^2}. \quad (2.11)$$

So that for a given AQL, k is determined by the sample size n .

III. NUMERICAL ILLUSTRATIONS

For optical evaluation the curves for OC functions are shown in Figure 1 for different value of known coefficient of variation. Table 1 is used for selection of known σ single sampling variables plan with known CV. For example, if the AQL is fixed at 1%, the AOQL is fixed at 0.125% and $v = 0, 4, 8, 12$ and 16 then Table 1 yields $n = 40, 39, 34, 27$ and 22 , and $k = 2.831, 2.869, 2.924, 2.969$ and 2.991 respectively. Further, suppose that it is decided to use σ , an acceptance criterion where σ is known to be 2.0. Let there exist an upper specific limit $U = 10.0$ and a unit for which the quality characteristic $x > U$ is considered as nonconforming. Table 2 shows the performance characteristics of a sampling plan with different values of n and k under a rectifying inspection scheme. If the true process average quality is operating at AQL ($\mu = 5.342$), then 95% of the lots submitted will be accepted during the sampling inspection stage itself and only 5% of the rejected lots will be rectified by replacing defective units with non defective units. The AOQ in such a case will not exceed the AOQL of 1.25% fixed, meaning that, irrespective of the product quality submitted by the producer, the consumer will receive an average quality not worse than 1.25% under the rectification scheme. When using Table 1 to select sampling plans, limitations of plans indexed by AOQL with known CV must be taken into account. Sampling with rectification of rejected lots reduces the average percentage of defective items in the lots; however, it also introduces non-homogeneity in the series of lots finally accepted. That is, any particular lot will have a quality of $p\%$ or 0% non-conforming depending on whether the lot is accepted or rectified. Thus, the assumption underlying the AOQL principle is that the homogeneity in the qualities of individual lots is unimportant and only average quality matters. From table 3 it is clear that the value of OC function with known CV shows higher values for the lot of bad quality in all the different cases ($v = 4, 8, 12, 16$) as compare to the independent case ($v = 0$).

Table-1: Single Sampling Plans for Variables Indexed by AQL and AOQL with Known CV

CV	AOQL(%)	AQL(%)												
		0.040	0.065	0.100	0.150	0.250	0.400	0.650	1.000	1.500	2.500	4.000	6.500	
v=0	0.050	47,3.112												
	0.080	17,2.900	46,2.972											
	0.125	9,2.754	16,2.807	40,2.831										
	0.200	6,2.608	9,2.661	14,2.652	30,2.670									
	0.320	4,2.464	6,2.523	8,2.500	12,2.489	31,2.513								
	0.500	3,2.324	4,2.392	5,2.361	7,2.338	12,2.327	31,2.355							
	0.800	2,2.168	3,2.249	4,2.213	4,2.183	6,2.154	11,2.145	29,2.178						
	1.250		2,2.104	3,2.065	3,2.031	4,1.994	6,1.968	10,1.961	25,1.994					
	2.000			2,2.006	2,1.860	3,1.817	4,1.783	5,1.748	8,1.741	18,1.788				
	8.000					2,1.623	2,1.585	3,1.551	4,1.529	6,1.522	17,1.561			
v=4	0.050	45,3.142												
	0.080	16,3.061	45,3.006											
	0.125	8,3.019	15,2.922	39,2.869										
	0.200	5,2.994	8,2.877	13,2.785	30,2.726									
	0.320	3,2.978	4,2.852	6,2.743	11,2.649	30,2.686								
	0.500	2,2.969	3,2.838	4,2.721	6,2.614	11,2.647	30,2.531							
	0.800	2,2.962	2,2.829	3,2.710	3,2.592	5,2.628	9,2.488	30,2.358						
	1.250		2,2.823	2,2.703	2,2.614	3,2.618	5,2.470	9,2.313	24,2.197					
	2.000			1,1.697	2,2.612	2,2.612	3,2.463	4,2.294	7,2.154	17,1.882				
	8.000					2,2.612	2,2.458	2,2.286	3,2.138	5,1.813	9,1.723			
v=8	0.050	109,3.195												
	0.080	78,3.166	105,3.055											
	0.125	71,3.157	77,3.029	34,2.924										
	0.200	68,3.154	71,3.021	9,2.897	24,2.792									
	0.320	67,3.152	68,3.017	5,2.891	6,2.771	25,2.632								
	0.500	66,3.151	67,3.015	3,2.889	4,2.768	6,2.611	24,2.476							
	0.800	66,3.150	66,3.014	2,2.888	2,2.765	3,2.606	7,2.457	22,2.306						
	1.250		66,3.013	2,2.887	2,2.765	2,2.605	4,2.452	7,2.288	19,2.145					
	2.000			1,2.887	1,2.764	2,2.604	3,2.451	4,2.284	5,2.128	11,1.980				
	8.000					2,2.604	2,2.449	2,2.282	3,2.125	4,1.970	12,1.772			
v=12	0.050	43,3.232												
	0.080	12,3.221	39,3.094											
	0.125	7,3.219	21,3.097	27,2.964										
	0.200	4,3.218	5,3.081	8,2.957	19,2.839									
	0.320	3,3.217	4,3.081	4,2.955	6,2.883	19,2.678								
	0.500	2,3.217	3,3.080	3,2.954	3,2.832	6,2.673	19,2.523							
	0.800	2,3.216	2,3.080	2,2.954	2,2.832	3,2.671	5,2.517	17,2.413						
	1.250		2,3.080	2,2.954	2,2.831	2,2.671	4,2.517	7,2.411	12,2.199					
	2.000			1,2.954	1,2.831	2,2.671	2,2.517	4,2.411	1,2.190	6,2.036				
	8.000					1,2.671	2,2.516	3,2.342	1,2.258	1,2.034	2,1.918			
v=16	0.050	28,3.255												
	0.080	6,3.251	22,3.117											
	0.125	5,3.251	7,3.115	22,2.991										
	0.200	3,3.251	5,3.114	8,2.989	18,2.868									
	0.320	2,3.250	4,3.114	4,2.988	5,2.866	9,2.706								
	0.500	2,3.250	3,3.114	2,2.988	3,2.866	5,2.705	9,2.551							
	0.800	2,3.250	2,3.114	2,2.988	2,2.865	3,2.705	3,2.550	7,2.382						
	1.250		2,3.113	1,2.988	1,2.865	2,2.705	3,2.550	6,2.382	4,2.224					
	2.000			1,2.988	1,2.865	2,2.705	2,2.550	4,2.382	4,2.224	2,2.068				
	8.000					1,2.705	2,2.550	2,2.381	2,2.224	1,2.067	7,1.858			
v=20	0.050	28,3.255												
	0.080	6,3.251	22,3.117											
	0.125	5,3.251	7,3.115	22,2.991										
	0.200	3,3.251	5,3.114	8,2.989	18,2.868									
	0.320	2,3.250	4,3.114	4,2.988	5,2.866	9,2.706								
	0.500	2,3.250	3,3.114	2,2.988	3,2.866	5,2.705	9,2.551							
	0.800	2,3.250	2,3.114	2,2.988	2,2.865	3,2.705	3,2.550	7,2.382						
	1.250		2,3.113	1,2.988	1,2.865	2,2.705	3,2.550	6,2.382	4,2.224					
	2.000			1,2.988	1,2.865	2,2.705	2,2.550	4,2.382	4,2.224	2,2.068				
	8.000					1,2.705	2,2.550	2,2.381	2,2.224	1,2.067	7,1.858			

Table-2: Performance Characteristics of the Variable Plan with Known CV

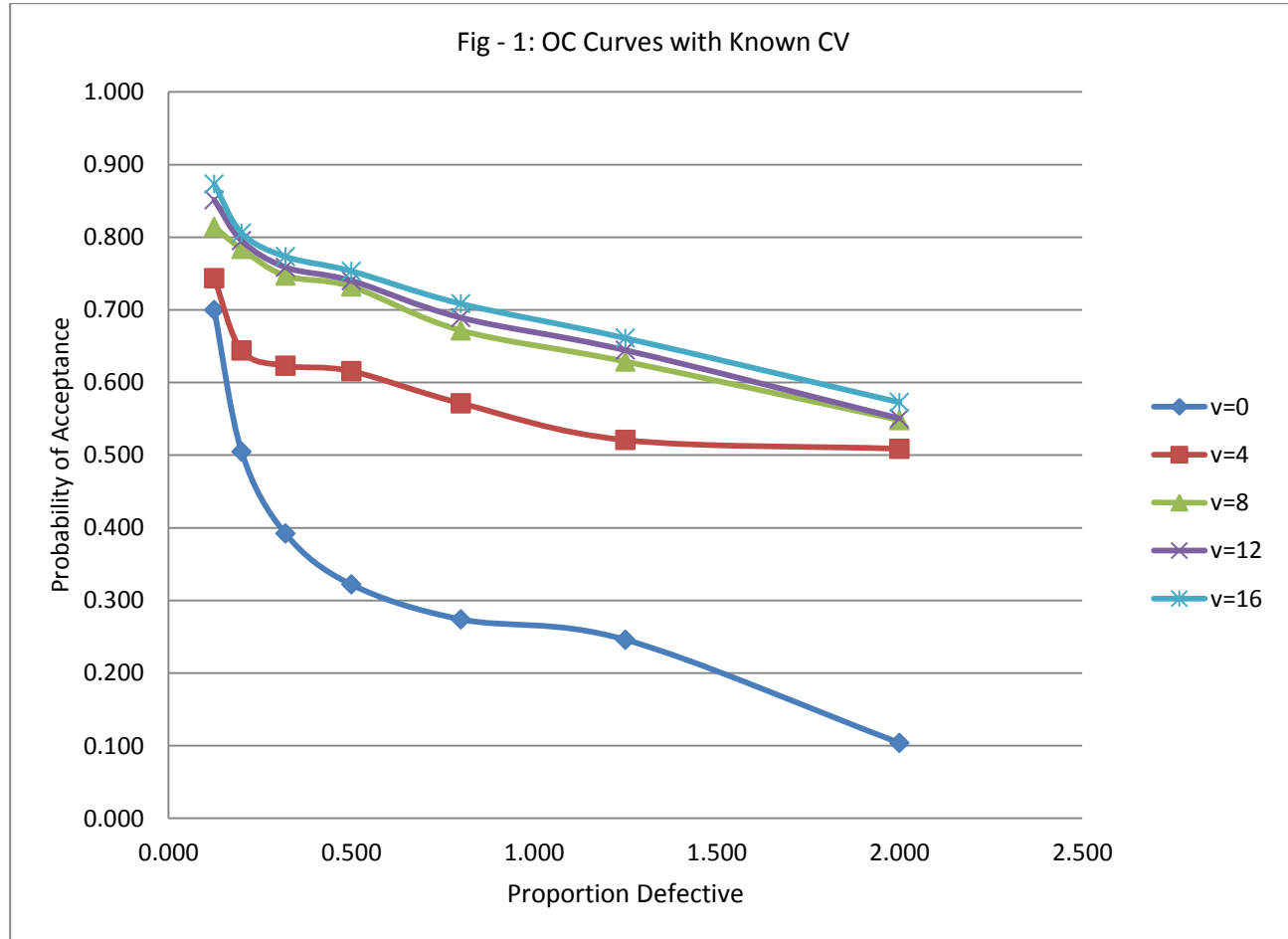
AQL = 0.001, AOQL = 0.00125, U = 10, S.D. = 2

CV	mew	v'	p(%)	w	Pa	AOQ
v=0	3.8200	3.0900	0.10	1.6381	0.9493	0.0950
	4.2000	2.9000	0.19	0.4364	0.6687	0.1248
	4.5000	2.7500	0.30	-0.5123	0.3042	0.0907
	5.0000	2.5000	0.62	-2.0934	0.0182	0.0113
	5.3000	2.3500	0.94	-3.0421	0.0012	0.0011
	5.6000	2.2000	1.39	-3.9908	0.0000	0.0000
v=4	3.8200	3.0900	0.10	1.6390	0.9494	0.0950
	4.2000	2.9000	0.19	0.2299	0.5909	0.1103
	4.5000	2.7500	0.30	-0.8825	0.1887	0.0562
	5.0000	2.5000	0.62	-2.7366	0.0031	0.0019
	5.3000	2.3500	0.94	-3.8490	0.0001	0.0001
	5.6000	2.2000	1.39	-4.9614	0.0000	0.0000
v=8	3.8200	3.0900	0.10	1.6433	0.9498	0.0951
	4.2000	2.9000	0.19	-0.2376	0.4061	0.0758
	4.5000	2.7500	0.30	-1.7225	0.0425	0.0127
	5.0000	2.5000	0.62	-4.1974	0.0000	0.0000
	5.3000	2.3500	0.94	-5.6823	0.0000	0.0000
	5.6000	2.2000	1.39	-7.1672	0.0000	0.0000
v=12	3.8200	3.0900	0.10	1.6477	0.9503	0.0951
	4.2000	2.9000	0.19	-0.8369	0.2013	0.0376
	4.5000	2.7500	0.30	-2.7984	0.0026	0.0008
	5.0000	2.5000	0.62	-6.0676	0.0000	0.0000
	5.3000	2.3500	0.94	-8.0291	0.0000	0.0000
	5.6000	2.2000	1.39	-9.9906	0.0000	0.0000
v=16	3.8200	3.0900	0.10	1.6387	0.9494	0.0950
	4.2000	2.9000	0.19	-1.5063	0.0660	0.0123
	4.5000	2.7500	0.30	-3.9893	0.0000	0.0000
	5.0000	2.5000	0.62	-8.1275	0.0000	0.0000
	5.3000	2.3500	0.94	-10.6104	0.0000	0.0000
	5.6000	2.2000	1.39	-13.0934	0.0000	0.0000

Table-3: Pa(pm) Values with Known CV

CV	AOQL(%)	AQL(%)											
		0.040	0.065	0.100	0.150	0.250	0.400	0.650	1.000	1.500	2.500	4.000	6.500
v=0	0.050	0.700											
	0.080	0.501	0.727										
	0.125	0.389	0.515	0.700									
	0.200	0.311	0.395	0.505	0.663								
	0.320	0.258	0.318	0.392	0.489	0.696							
	0.500	0.226	0.270	0.322	0.389	0.510	0.714						
	0.800	0.203	0.236	0.274	0.321	0.402	0.514	0.719					
	1.250		0.227	0.246	0.281	0.338	0.413	0.530	0.702				
	2.000			0.104	0.255	0.297	0.349	0.425	0.524	0.672			
	3.200					0.276	0.313	0.365	0.428	0.514	0.696		
	5.000							0.336	0.379	0.435	0.540	0.714	
	8.000								0.358	0.395	0.460	0.553	0.720

v=4	0.050	0.618																		
	0.080	0.613	0.739																	
	0.125	0.590	0.633	0.743																
	0.200	0.571	0.610	0.644	0.660															
	0.320	0.565	0.597	0.623	0.645	0.741														
	0.500	0.556	0.556	0.616	0.630	0.666	0.744													
	0.800	0.533	0.485	0.571	0.635	0.650	0.684	0.746												
	1.250		0.446	0.521	0.568	0.635	0.667	0.698	0.715											
	2.000			0.509	0.513	0.559	0.581	0.689	0.710	0.748										
	3.200					0.439	0.559	0.651	0.709	0.718	0.738									
	5.000								0.532	0.679	0.665	0.698	0.786							
8.000									0.606	0.639	0.576	0.767	0.801							
v=8	0.050	0.760																		
	0.080	0.704	0.802																	
	0.125	0.689	0.716	0.814																
	0.200	0.656	0.626	0.784	0.811															
	0.320	0.617	0.601	0.747	0.806	0.821														
	0.500	0.566	0.589	0.732	0.764	0.808	0.831													
	0.800	0.552	0.521	0.672	0.756	0.799	0.779	0.840												
	1.250		0.478	0.629	0.713	0.755	0.717	0.779	0.835											
	2.000			0.548	0.651	0.661	0.606	0.694	0.798	0.847										
	3.200					0.532	0.574	0.666	0.739	0.790	0.837									
	5.000								0.579	0.702	0.751	0.793	0.874							
8.000									0.625	0.720	0.718	0.798	0.887							
v=12	0.050	0.782																		
	0.080	0.754	0.813																	
	0.125	0.693	0.785	0.851																
	0.200	0.662	0.745	0.795	0.848															
	0.320	0.620	0.675	0.759	0.820	0.861														
	0.500	0.575	0.613	0.740	0.784	0.819	0.861													
	0.800	0.567	0.548	0.689	0.764	0.807	0.824	0.879												
	1.250		0.493	0.645	0.719	0.757	0.743	0.780	0.883											
	2.000			0.551	0.661	0.678	0.675	0.696	0.901	0.894										
	3.200					0.571	0.584	0.605	0.899	0.886	0.894									
	5.000							0.498	0.877	0.840	0.832	0.909								
8.000								0.813	0.815	0.814	0.818	0.909								
v=16	0.050	0.865																		
	0.080	0.849	0.883																	
	0.125	0.784	0.840	0.873																
	0.200	0.748	0.756	0.806	0.855															
	0.320	0.683	0.680	0.773	0.828	0.904														
	0.500	0.638	0.650	0.753	0.791	0.832	0.904													
	0.800	0.612	0.584	0.708	0.776	0.812	0.865	0.912												
	1.250		0.531	0.661	0.726	0.762	0.779	0.808	0.921											
	2.000			0.573	0.674	0.686	0.678	0.715	0.821	0.924										
	3.200					0.583	0.607	0.632	0.769	0.907	0.974									
	5.000								0.550	0.625	0.883	0.882	0.923							
8.000									0.518	0.868	0.865	0.866	0.913							



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