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An EOQ Model with Partial Backorder for Fuzzy Demand and Learning in Fuzziness

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Abstract— This study demonstrates an EOQ model with partial backorders over the finite time horizon assuming imprecise demand which is characterized by triangular fuzzy number. Learning effect is considered to reduce the impreciseness of demand as inventory planners get experienced by collecting knowledge from previous cycles. This paper aims to find out the optimal number of replenishments and an optimal fraction of the cycle during the positive inventory to minimize the total annual cost. The optimal policy is derived using analytical approach for crisp and fuzzy model whereas algorithmic procedure is adopted for the fuzzy-learning model. To show the significance of learning effect, numerical analysis is executed and compared results from the crisp, fuzzy and fuzzy-learning case which shows that increasing human learning reduces fuzziness of the demand and approaches to the crisp model.

Keywords— EOQ model, partial backordering, fuzzy demand, learning in fuzziness, centroid method

I. INTRODUCTION

Since Harris [1] presented a fundamental EOQ model, a lot of extensions of it in different directions are presented to portray the real situation. Interested readers may refer Glock et al. [2]'s comprehensive review for the lot-sizing problems. Many researchers relaxed the assumption 'No shortages' of the basic EOQ model. As many organisation has to deal with shortages, where some customers are willing to wait and some may not. Hence, a model with partial backordering came into an account. Pentico and Drake [3] presented a survey of the EOQ and EPQ with partial backordering in the crisp environment. Fabrycky and Banks [4] and Ali [5] were first to investigate an EOQ model with partial backorders, but no solution procedure was derived by them. Montgomery et al. [6] demonstrated a lot size model with partial backorders describing solution procedure. Additionally, it is mandatory to fix the planning horizon for many organisation such as accounting, finance or risk management so that other options may be estimated for better performance during the same period. Dye et al. [7] presented a lot size model with partial backorders and inserted cost for lost sale and the purchase cost of backordered units under the finite time horizon. Afterwards, lots of literature with partial backorders under the finite time horizon such as Zhou et al. [8], You and Wu [9], Uthayakumar and Geetha [10], Yang [11], Wee [12], Ouyang et al. [13], Chang et al. [14], Tsao [15] etc. are contributed to the literature by various researchers.

In the real-life situation, demand cannot be deterministic always and inventory planner has to face uncertainty. Stochastic techniques are serving for uncertainty since long. But it is not enough to deal with uncertainty as it relies on past data. The fuzzy set theory developed by Zadeh [16] is the powerful tool to deal with uncertainty which can convert linguistic expressions to a mathematical expression. The fuzzy set theory was first applied by Park [17] on an EOQ model. Vujosevic et al. [18] presented an inventory model with shortages by taking imprecise demand and cost parameters. Researchers such as Yadav et al. [19], Ouyang and Chang [20], Chang et al. [21], Chang et al. [22], Yang et al. [23], Wang et al. [24], Buckley et al. [25] considered partial backorders in to develop inventory model with fuzzy demand. De and Goswami [26], Maity and Maiti [27], Yazgı Tütüncü et al. [28] etc. developed inventory model under finite time horizon by considering shortages. Shekarian et al. [29] etc. presented a comprehensive review on fuzzy inventory models.

In everyday life, human keeps learning with experience over the time. In any business, demand is uncertain initially, then the inventory planner starts getting knowledge and ideas about market demand and customer preferences during the planning horizon. Hence, ambiguity for demand estimation gets reduced. In this paper, we use this characteristic to reduce imprecision of fuzzy demand. There is a vast literature of this characteristic viz. 'learning curve' available (cf. [30], [31] etc.). Few researchers have contributed to the literature on inventory models incorporating the learning effect. Glock et al. [32] referred in the paper that reliable and updated information can be fetched with the experience over the time as an inventory planner accumulates more knowledge with experience over the planning period. Researchers such as Kazemi et al. [33], Kazemi et al. [34] and Soni et al. [35] incorporated human learning to reduce imprecision of the model parameters.

In this paper, we study an EOQ model with partial backlogging under the finite time horizon which is the extended work of Glock et al. [32]. The main objective of this paper is to find optimal replenishment and the optimal time fraction when positive inventory so that total cost become minimum. The solution procedure is presented in a crisp, fuzzy and fuzzy-learning environment. There is no such investigation for inventory model with partial backorders carried out in the present inventory literature. The remaining part of the paper consists of the following section. Notations and assumptions are presented in Section 2. We demonstrate the mathematical model in Section 3. To examine the model, we present the numerical examples and sensitivity analysis in Section 4, whereas the conclusion and future scope are given in Section 5.

II. NOTATIONS AND ASSUMPTIONS

We use following notation throughout the model.

2.1 Notations

D	:	The constant demand rate per unit time.
A	:	The ordering cost per order.
h	:	Holding cost per unit per unit time.
s	:	Shortage cost per unit per year
π	:	Lost sale cost per unit per year
Q	:	The order quantity.
H	:	The length of the finite planning horizon
T	:	The length of each replenishment cycle
$I(t)$	:	The inventory level at time t
n	:	Number of orders (Decision variable and is an integer), $n=\frac{H}{T}$
F	:	The fraction of the replenishment cycle where the net inventory level is positive (Decision variable)
$TC(F,n)$	:	The total cost over finite time horizon H.

2.2 Assumptions

(1) Inventory system considers a single item over the finite period H.

(2) The rate of demand is taken as imprecise and characterised by triangular fuzzy number $\tilde{D}=(D-\Delta_1, D, D+\Delta_2)$ where, $0<\Delta_1<D$ and $0<\Delta_2<D$ where Δ_1 and Δ_2 are imprecision parameters of demand and determined by a decision maker.

(3) Shortages are allowed and partially backlogged where B fraction of demand is backlogged, $0 \le B \le 1$.

(4) Replenishment rate is infinite and lead time is zero.

III. MODEL FORMULATION

3.1 Crisp inventory model

The total spell H is divided into n equal intervals of length T. Hence, $T=H/n$; n is an integer. So, time periods for ordering cycles over the finite plan horizon H are $T_j = jT$, j = 0, 1, …, n. Inventory starts with maximum inventory and inventory level $I(t)$ at time t decreases due to the demand and becomes zero at the time $t_j\ (j=1..n)$. Shortages start to be accumulated at this point of time and become maximum at $T_j\ (j=1..n)$. The model considers that the shortages are partially backlogged by B fraction of demand and remaining sales are lost. The inventory system is depicted in Figure 1 where inventory run time $t_j\ (j=1..n)$ is presented in terms of the number of orders (n) and the fraction of ordering cycle with positive stock (F) (cf. [36]). The inventory system is depicted in Fig.1.

With all the above description, the total cost in the time horizon $[0,H]$ is given by

$$\begin{aligned} TC(F,n) &= \text{ordering cost + holding cost} \\ &\quad \text{+ shortage cost+lost sales cost} \\ &= (n+1)A + DH \times \\ &\quad \left[\frac{H}{2n}\left(sB(1-F)^2 + hF^2\right) + \pi(1-F)(1-B)\right] \end{aligned} \tag{1}$$

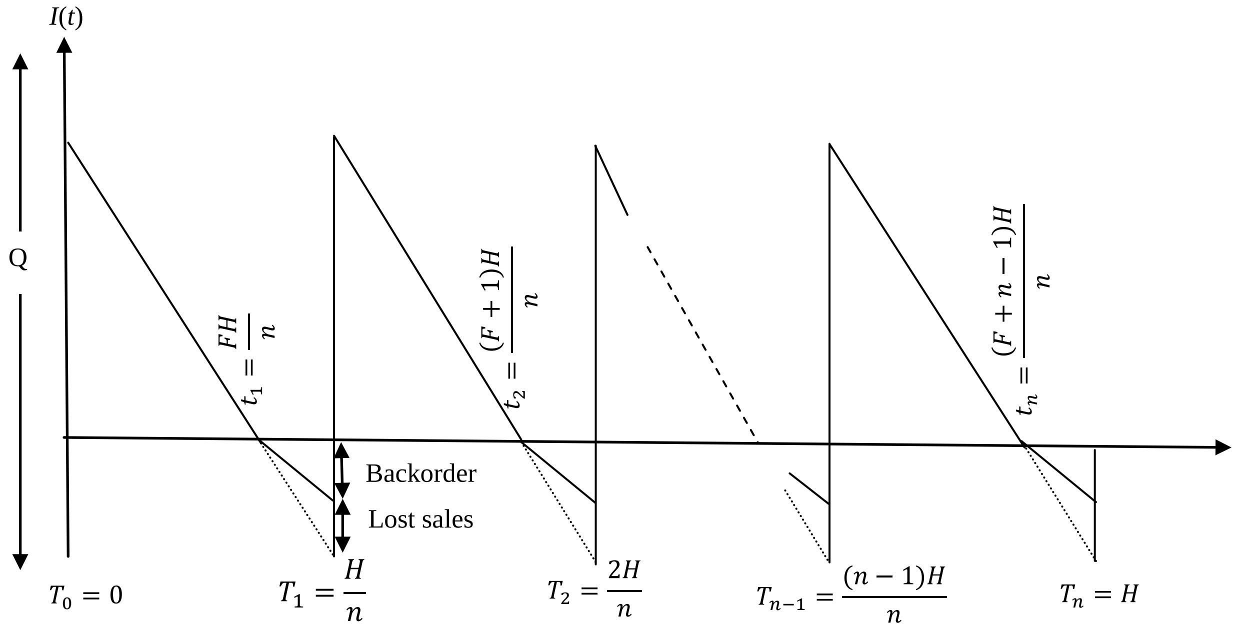

Figure 1: Graphical representation of the inventory system

Now, the main aim is to find an optimal number of orders and the optimal fraction of inventory when positive inventory so that $TC(F,n)$ presented in (1) reaches to its minimum value. By using an analytic method, we set first partial derivatives with respect to F and n equal to zero in order to find their optimal values which are given by

$$\frac{\partial TC(F,n)}{\partial F} = \frac{DH^2}{n}\left[-sB(1-F)+hF\right] - \pi DH(1-B) = 0 \quad (2)$$

And $$\frac{\partial TC(F,n)}{\partial n} = A - \frac{DH^2}{2n^2}\left[sB(1-F)^2 + hF^2\right] = 0 \quad (3)$$

By using (2) and (3), we get the expression for optimal values F_c^* and n_c in a crisp environment as

$$F_c^* = \frac{sBH + \pi n(1-B)}{(h+Bs)H} \quad \text{and}$$

$$n_c = H \times \sqrt{\frac{D\left(Bs(1-F)^2 + hF^2\right)}{2A}}$$

By substituting F_c^* into n_c, we get

$$n_c = H \times \sqrt{\frac{hsBDH(X-DY)}{(X-DY)}}; \quad (4)$$

where, $X = 2A(Bs+h), Y = \pi^2(1-B)^2$

Therefore, the optimal number of replenishment n_c^* can be derived using the condition

$$TC\left(F_c^*, n_c^*\right) = \min\left\{TC\left(F_c^*, \lfloor n_c \rfloor - 1\right), TC\left(F_c^*, \lfloor n_c \rfloor\right), TC\left(F_c^*, \lfloor n_c \rfloor + 1\right)\right\}$$

where $\lfloor x \rfloor$ = integer part of x.

Hence, a formula to find out n_c^* is given by

$$n_c^* = \text{Round integer } H \times \sqrt{\frac{hsBDH(X-DY)}{(X-DY)}}; \quad (5)$$

where, $X = 2A(Bs+h), Y = \pi^2(1-B)^2$

Now, to prove $TC(F,n)$ is minimum at (F_c^*, n_c^*), taking second-order partial derivatives of $TC(F,n)$ with respect to F and n, we get

$$\frac{\partial^2 TC(F,n)}{\partial F^2} = \frac{DH^2(h+Bs)}{n} > 0$$

$$\frac{\partial^2 TC(F,n)}{\partial n^2} = \frac{DH^2}{n^3}\left[sB(1-F)^2 + hF^2\right] > 0$$

$$\frac{\partial^2 TC(F,n)}{\partial nF} = \frac{\partial^2 TC(F^*,n)}{\partial Fn} = \frac{-DH^2}{n^2}\left[-sB(1-F)+hF\right]$$

Hence, the relevant determinant of the Hessian matrix is given by

$$\begin{vmatrix} \frac{\partial^2 TC(F,n)}{\partial F^2} & \frac{\partial^2 TC(F,n)}{\partial n \partial F} \\ \frac{\partial^2 TC(F,n)}{\partial F \partial n} & \frac{\partial^2 TC(F,n)}{\partial n^2} \end{vmatrix} = \frac{D^2 H^4 Bhs}{n^4} > 0 \tag{6}$$

Therefore, $TC(F,n)$ attains its minimal value and optimal solution (F_c^*, n_c^*) in a crisp environment

Additionally, optimal order quantity is given by

$$Q_c^* = \frac{DH\left(B\left(1-F_c^*\right)+F_c^*\right)}{n_c^*} \tag{7}$$

3.2 Inventory model under fuzzy environment

In this section, we relax the assumption that demand is precise in order to portray the real situation. Demand rate $\tilde{D}$ is characterised as triangular fuzzy number (as defined in assumption). Hence, the objective function given in (1) is transformed into a triangular fuzzy number viz. $TC(F,n) = \left(TC_1(F,n), TC_2(F,n), TC_3(F,n)\right)$, where $TC_i(F,n)(i=1,2,3)$ are real-valued functions and satisfy the condition $TC_1(F,n) \le TC_2(F,n) \le TC_3(F,n)$. Moreover $TC_i(F,n)(i=1,2,3)$ can be presented using function principle (cf. [37]) as

$$TC_1(F,n) = (n+1)A + (D-\Delta_1)H \times \left[\frac{H}{2n}\left(sB(1-F)^2 + hF^2\right) + \pi(1-F)(1-B)\right]$$

$$TC_2(F,n) = (n+1)A + DH \times \left[\frac{H}{2n}\left(sB(1-F)^2 + hF^2\right) + \pi(1-F)(1-B)\right]$$

$$TC_3(F,n) = (n+1)A + (D+\Delta_2)H \times \left[\frac{H}{2n}\left(sB(1-F)^2 + hF^2\right) + \pi(1-F)(1-B)\right]$$

Now, to obtain a crisp equivalent form of the fuzzy objective function, we employ centroid formula and express as

$$\begin{aligned} M\left(TC(F,\ n)\right) &= \frac{1}{3}\left[TC_1 + TC_2 + TC_3\right] \\ &= TC(F,\ n) + \frac{H(\Delta_2 - \Delta_1)}{3} \times \\ &\left[\frac{H}{2n}\left(sB(1-F)^2 + hF^2\right) + \pi(1-F)(1-B)\right] \end{aligned} \tag{8}$$

Optimal solutions of the fuzzy objective function given in (8) can be derived in the way as in crisp case and given by

$$F_F^* = \frac{sBH + \pi n(1-B)}{(h+Bs)H} \text{ and } n_F^* = \text{Round integer } H \times \sqrt{\frac{\left(Bs\left(1-F_F^*\right)^2 + hF_F^{*2}\right)\left(3D + \Delta_2 - \Delta_1\right)}{6A}} \tag{9}$$

Where F_F^* is the optimal fraction of inventory when there is positive inventory in the fuzzy environment and n_F^* is an optimal number of replenishments. Integer value for the optimal number of replenishments n_F^* satisfies the following condition

$$\begin{aligned} M\left(TC\left(F_F^*, n_F^*\right)\right) = \min\Big\{ & M\left(TC\left(F_F^*, \lfloor n_F \rfloor - 1\right)\right), \\ & M\left(TC\left(F_F^*, \lfloor n_F \rfloor\right)\right), M\left(TC\left(F_F^*, \lfloor n_F \rfloor + 1\right)\right)\Big\} \end{aligned} \tag{10}$$

Similarly, it is easy to show (F_F^*, n_F^*) is a minimal to the function $M\left(TC(F,\ n)\right)$ with respect to F and *n*. Hence (F_F^*, n_F^*) is the optimal solution.

Consequently, the optimal order quantity in the fuzzy sense is given by

$$Q_F^* = \frac{DH\left(B\left(1-F_c^*\right)+F_c^*\right)}{n_c^*} + \frac{H\left(B\left(1-F_c^*\right)+F_c^*\right)\left(\Delta_2 - \Delta_1\right)}{3n_c^*} \tag{11}$$

3.3 Inventory model with learning effect to the fuzziness

Human has a characteristic to learn with repetitive task over the planning horizon. In order to use this phenomenon in the mathematical model, we use the "learning curve" which is the mathematical representation of the rate at which decision maker uses his/her cumulative experience to improve the work. In this paper, it is assumed that learning of a decision maker reduces the fuzziness of demand as s/he accumulates knowledge about the market demand by repetition of placing orders over the planning horizon. Although many learning curves developed so far (cf. [38]), we use a popular and widely used power learning curve presented by Wright [39] which is of the form $y_x = y_1 * x^{-b}$ where y_x is performance at the time of x^{th} order, y_1 is the performance at the starting cycle and *b* is learning exponent.

Decision maker gets more knowledge by repetition of placing orders over the time. If the fuzziness parameters Δ_1 and Δ_2 are affected by learning and same learning rate applied for both parameters then the fuzziness parameter $j,\ j = 1,\ 2$, at the time of k^{th} replenishment as given by Glock et al. [32] is

$$\Delta_{j,k}=\begin{cases}\Delta_{j,1} & k=1\\ \Delta_{j,1}\left(\dfrac{(k-1)H}{n}\right)^{-l} & k>1\end{cases} \tag{12}$$

The total annual cost after applying human learning for a k^{th} replenishment cycle is given by

$$\frac{(n+1)A}{n}+\frac{DH}{n}\left[\frac{H}{2n}\left(sB(1-F)^2+hF^2\right)+\pi(1-F)(1-B)\right]+\frac{H}{3n}\left[\frac{H}{2n}\left(sB(1-F)^2+hF^2\right)+\pi(1-F)(1-B)\right]\times\left(\Delta_{2,1}-\Delta_{1,1}\right)\left(\frac{(k-1)H}{n}\right)^{-l} \tag{13}$$

where $1<k\le n$ and $n\ge 2$.

Hence, the total cost for n replenishments under the effect of learning in fuzziness is given by

$$M\left(TC_{FL}(F,n)\right)=(n+1)A+DH\times\left[\frac{H}{2n}\left(sB(1-F)^2+hF^2\right)+\pi(1-F)(1-B)\right]+\frac{H}{3n}\left[\frac{H}{2n}\left(sB(1-F)^2+hF^2\right)+\pi(1-F)(1-B)\right]\left(\Delta_{2,1}-\Delta_{1,1}\right)\left[1+\sum_{k=2}^{n}\left(\frac{(k-1)H}{n}\right)^{-l}\right] \tag{14}$$

Now, by setting the necessary condition $\partial M\left(TC_{FL}(F,n)\right)/\partial F=0$, we derive the optimal fraction of time when inventory is positive as $F^*_{FL}=\dfrac{sBH+\pi n^*_{FL}(1-B)}{(h+Bs)H}$.

Also, by sufficient condition with respect to F,

$$\partial^2 M\left(TC_{FL}(F,n)\right)/\partial F^2=\frac{dH^2(h+Bs)}{n}+\frac{1}{3n^2}\left(H^2(h+Bs)\left(\Delta_{2,1}-\Delta_{1,1}\right)\left[1+\sum_{k=2}^{n}\left(\frac{(k-1)H}{n}\right)^{-l}\right]\right)>0$$

Hence, $M\left(TC_{FL}(F,n)\right)$ is strictly convex in F.

Optimality of $M\left(TC_{FL}(F,n)\right)$ with respect to n cannot be derived easily by the standard analytic method as n is appeared in the summation in (14). Hence the optimal number of orders $\left(n^*_{FL}\right)$ in the fuzzy sense with learning is derived by the algorithmic procedure.

Algorithm

Step 1:

Substitute $F_{FL}=\dfrac{sBH+\pi n(1-B)}{(h+Bs)H}$ in $M\left(TC_{FL}(F,n)\right)$

Choose an initial trial value of n^*_{FL}, say $n=\lfloor n_F\rfloor$ and compute $M\left(TC_{FL}(F^*_{FL},n)\right)$ and $M\left(TC_{FL}(F^*_{FL},n-1)\right)$.

Step 2:

If $M\left(TC_{FL}(F^*_{FL},n)\right)\ge M\left(TC_{FL}(F^*_{FL},n-1)\right)$, then compute $M\left(TC_{FL}(F^*_{FL},n-2)\right), M\left(TC_{FL}(F^*_{FL},n-3)\right)$..., until we find $M\left(TC_{FL}(F^*_{FL},k)\right)<M\left(TC_{FL}(F^*_{FL},k-1)\right)$. Set $n^*_{FL}=k$ and stop.

Step 3:

If $M\left(TC_{FL}(F^*_{FL},n)\right)<M\left(TC_{FL}(F^*_{FL},n-1)\right)$, then compute $M\left(TC_{FL}(F^*_{FL},n+1)\right), M\left(TC_{FL}(F^*_{FL},n+2)\right)$..., until we find $M\left(TC_{FL}(F^*_{FL},k)\right)<M\left(TC_{FL}(F^*_{FL},k+1)\right)$. Set $n^*_{FL}=k$ and stop.

The optimal order quantity in the fuzzy sense with learning to fuzziness is given by

$$Q^*_{FL}=\frac{DH\left(B\left(1-F^*_c\right)+F^*_c\right)}{n^*_{FL}}+\frac{H\left(B\left(1-F^*_c\right)+F^*_c\right)}{3n^*_{FL}}\times\left(\Delta_{2,1}-\Delta_{1,1}\right)\left(\frac{\left(n^*_{FL}-1\right)H}{n^*_{FL}}\right)^{-l} \tag{15}$$

IV. NUMERICAL EXAMPLE

In this section, we study the numerical analysis to examine applicability of the model with learning effect. We consider the following estimation of parameters: D = 200 units/month, A = \$50/order, h = \$3/unit/month, s = \$1/unit/month, B = 0.5, π = 2/unit/month, H = 12. l = 0.322, i.e. 80% learning rate, $(\Delta_1,\Delta_2)=(20,100)$, and $(\Delta_1,\Delta_2)=(2,8)$. Using the solution procedure demonstrated in the earlier section, outcomes are presented in Table 1.

Table 1 shows that the optimal total cost $TC\left(F^*,n^*\right)$ and the optimal number of orders $\left(n^*\right)$ reduce by considering learning in the fuzziness of the demand. It is clear that human learning reduces the ambiguity and gain knowledge about consumer's choice and hence market demand. Inventory planner's learning about demand reduces the fuzziness of the demand. It suggests ordering high demand in later shipments with compare to earlier shipments. Ex. 2 is demonstrated to examine the optimal policy for the lower fuzziness of the demand which incurs the lower cost but invariant in $\left(n^*\right)$. The fraction of the replenishment cycle when inventory level is positive $\left(F^*\right)$ is also reduces while incorporating human learning. Table 2 clearly indicates that increasing learning effect from slow learning (*l*=100%) to fast learning (*l*=50%) results in reducing cost and eventually approaches to crisp model. Additionally, F^*_{FL} and n^*_{FL} decrease with the increase in human learning rate which suggests ordering less if the learning rate is more because the decision maker learns with time. Hence it is beneficial to order in fewer numbers in later cycles.

Table 1: Comparison of optimal solutions for a crisp, fuzzy and fuzzy-learning model

	EOQ		EOQ_F		EOQ_{FL}	
	Ex. 1	*Ex. 2*	*Ex. 1*	*Ex. 2*	*Ex. 1*	*Ex. 2*
	Δ_1=20 Δ_2=100	Δ_1=2 Δ_2=8	Δ_1=20 Δ_2=100	Δ_1=2 Δ_2=8	$\Delta_{1,1}$=20 $\Delta_{1,2}$=100	$\Delta_{1,1}$=2 $\Delta_{1,2}$=8
F^*	0.5476	0.5476	0.6190	0.5476	0.5952	0.5476
n^*	17	17	20	17	19	17
Q^*	109.24	109.24	110.10	110.34	109.42	109.95
$TC(F^*, n^*)$	2834.45	2834.45	3083.52	2853.80	2997.38	2846.95

Table 2: Comparison of optimal solutions for a crisp, fuzzy and fuzzy-learning model

l	*Ex. 1:* $(\Delta_1,\Delta_2)=(20,100)$				*Ex. 2:* $(\Delta_1,\Delta_2)=(2,8)$			
	F^*_{FL}	n^*_{FL}	$TC_F\left(F^*_F, n^*_F\right)$	Q^*_{FL}	F^*_{FL}	n^*_{FL}	$TC_{FL}\left(F^*_{FL}, n^*_{FL}\right)$	Q^*_{FL}
0.000(100%)	0.6190	20	3083.52	110.10	0.5476	17	2853.80	110.34
0.074(95%)	0.6190	20	3059.10	108.77	0.5476	17	2851.83	110.22
0.152(90%)	0.5952	19	3036.55	111.57	0.5476	17	2850.04	110.12
0.234(85%)	0.5952	19	3016.06	110.45	0.5476	17	2848.42	110.03
0.322(80%)	0.5952	19	2997.38	109.42	0.5476	17	2846.95	109.95
0.415(75%)	0.5952	19	2980.78	108.51	0.5476	17	2845.63	109.87
0.737(60%)	0.5714	18	2940.60	110.59	0.5476	17	2842.51	109.70
1.000(50%)	0.5714	18	2921.75	109.54	0.5476	17	2841.04	109.62

V. CONCLUSION

In this paper, we presented an EOQ model by incorporating partial backorder to the work presented by Glock et al. [32] assuming imprecise demand. The model assumes that human learning reduces the ambiguity of the fuzzy demand by placing an order over the finite time horizon. A mathematical model is presented along with its solution procedure to find optimal policies. Results, derived from numerical illustration, compared for all three models in a crisp, fuzzy and fuzzy-learning sense. Results from numerical studies reveal that optimal total cost is more in the fuzzy environment than the cost incurs by crisp model. It is clear from the outcomes that human learning is an important phenomenon and affects the fuzziness of fuzzy parameters of demand. As human learns with experience by placing orders over the time, fuzziness reduces with time and eventually approaches to the optimal policy derived by the crisp model. In the case of very fast learning, fuzzy learning model becomes almost similar to the crisp inventory model. Hence, human learning is a sensitive phenomenon and cannot be avoided for the decision-making process in any business. Model suggest to order a smaller quantity in faster learning as optimal inventory run time while positive inventory and the optimal number of orders reduce. Hence, the benefits of accumulated knowledge from previous experience can be added in the succeeding replenishments. In case of lower fuzziness of demand, optimal replenishment policy does not change by learning, still model suggests to order a smaller quantity which reduces optimal total cost. This study is applicable to the firms who face uncertainty problems for demand in the competitive marketplaces where some customers are not willing to wait and go for other alternates. This situation arises especially in case of launching a new product or brand where fuzzy theory serves as a powerful tool to deal with uncertainty. Decision makers start with the expert opinion and then learn from placing orders over the time and eventually, reduces fuzziness of demand.

Furthermore, this work can be extended in many directions. It can be extended by incorporating deterioration. Additionally, the main limitation of this paper is that forgetting is not considered as any organisation has to face forgetting effect when there is a long period between two consecutive orders. Hence, forgetting is an unforgettable phenomenon to be applied to this model. Also, the situation may arise with a combination of constant fuzziness and variable by learning in fuzziness or it may be a combination of motor and cognitive learning. Different learning curve presented by Grosse et al. [38] can be applied to consider the effect of human learning.

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