

Prediction of Flexural Strength of Soilcrete Blocks Using Scheffe's Simplex Lattice Design

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Abstract— The major constituent material in block production which is river sand, is costly, scarce and unavailable in some places. This has serious effect on the production cost of blocks used in construction. Flexural strength which is an important mechanical property has been ignored to an extent with regards to block moulding technology. This work presents the flexural strength of soilcrete blocks made with readily available and affordable late rite and the optimization of the flexural strength using Scheffe's simplex lattice design method. Statistical tools were used to verify the proposed optimization technique. The optimum value of the flexural strength is 1.452N/mm².

Index Term—Optimisation, Flexural Strength, Soilcrete Blocks, Scheffe's Simplex Method

I. Introduction

Soilcrete blocks are masonry units made of cement, late rite and water. Blocks are mostly used as walling units in the construction of shelter (which is one of the basic needs of man) and other infrastructures. The most commonly used material for block making in Western countries is river sand. It has been observed that there is an increasing rise in the cost of river sand and in fact river sand is unavailable in some places. This affects the production cost of blocks and consequently has made housing units unaffordable for middle class citizens of West African countries. An alternative material like laterite is readily available and affordable in most parts of West African countries.

The constituent materials of blocks should be mixed in their right proportions in order to achieve the desired strength. Various problems are associated with the traditional methods of mix design. To minimise some of the problems, an optimisation process has been proposed using Scheffe's simplex lattice design method.

Consequently, this research work deals with production of soilcrete blocks using readily available and affordable laterite, determination of the flexural strength and the optimization of flexural strength using Scheffe's method. The model developed for optimization of flexural strength of sand-laterite blocks was tested for adequacy using statistical tools. They all agreed to the acceptance of the model equation. With the model developed, a user can specify a desired value of flexural strength and there will be a print out of all possible mix ratios that will give that flexural strength. On the other hand, if an input of the mix ratio is made, the flexural strength comes out as the output. With the

optimization model, results are obtained in less time, with less effort and energy.

II. Related Works

Several researchers have worked on late rite blocks [1] – [4]. However, they tested for the compressive strength of the blocks. Some other important characteristics of blocks like the flexural strength have, to an extent been ignored. Flexural strength is the property of a solid that indicates its ability to resist failure in bending. It is also the strength of the block measured by subjecting it to flexure. The theoretical maximum tensile stress reached in the bottom fibre of the test specimen is known as the flexural strength [5]. The knowledge of this strength is of value in estimating the load under which cracking will develop. The absence of cracking is of considerable importance in maintaining the continuity of a structure.

Scheffe's simplex lattice design method has been applied successfully by various authors. Ezech, et.al. [6] optimized the compressive strength of laterite/sand hollow block using Scheffe's simplex method. Mama and Osadebe formulated models for prediction of compressive strength of sandcrete blocks using Scheffe's and Osadebe's optimization theories [7]. Orié developed models for optimization of compressive and flexural strength of mound soil concrete using Scheffe's method [8]. Okere et. al. worked on concrete mixture design and generated a model for optimization of concrete cube strength using Scheffe's optimization theory [9]. Obam developed a model for optimization of strength of palm kernel shell aggregate concrete using Scheffe's simplex theory [10]. It is worthy of note here that authors formulated models for compressive strength of concrete, sandcrete blocks and laterite/sand hollow blocks.

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III. Solution/Need/Importance of the study Problem Statement/Objectives

The major constituent material in block production which is river sand, is costly, scarce and unavailable in some places. This has serious effect on the production cost of blocks used in construction. Flexural strength which is an important mechanical property has been ignored to an extent with regards to block moulding technology. Various mix design methods have been developed in order to achieve the desired property of concrete/blocks. It has also been observed that these methods have some limitations. They are not cost effective and time and energy are spent in order to get the appropriate mix proportions. Hence there is need to optimize flexural strength of soilcrete blocks made with readily available and affordable laterite.

IV. Hypothesis

The model equation was tested for adequacy against the controlled experimental results. The statistical hypothesis for this mathematical model is as follows:

Null Hypothesis (H_0): There is no significant difference between the experimental and the theoretically expected results at an α -level of 0.5.

Alternative Hypothesis (H_1): There is a significant difference between the experimental and theoretically expected results at an α -level of 0.05.

V. Methodology

Experimental and analytical methods were used in this work. The following materials were used for the experimental investigation.

Laterite: This was sourced from Ikeduru L.G.A. Imo State. The grading and properties conformed to BS 882 [11].

Cement: Eagle cement brand of OPC with properties conforming to British standard.

Water: Potable water conforming to the specification of EN 1008 [12]

Analytical method

The analytical method was developed by Scheffe [13]. A theory is developed for experiments with mixtures of q -components whose purpose is the empirical prediction of the response to any mixture of the components, when the response depends only on the proportion of the component and not on the total amount. Scheffe introduced the (q,m) simplex lattice designs. Simplex is simply the projection of a q -dimensional space onto a $q-1$ dimensional coordinate system; this can be done because the proportions of the mixture are constrained to sum to one. Thus, feasible

combination of three components in this case (laterite, cement and water) can be projected onto a two-dimensional triangular field. The lattice part of the simplex lattice design shows that points are spaced regularly on the simplex. The degree of the simplex lattice is defined by the degree of the polynomial that may be used to fit the response surface over the simplex. Scheffe showed that the number of points in (q,m) lattice is given by

$${}^{q+m-1}C_m = q(q+1)\dots\dots(q+m-1)/m! \quad (1)$$

Hence for a three-component mixture, i.e. $(3,2)$ lattice, the number of points equals $3(3+1)/2! = 6$.

The (q,m) simplex lattice designs are characterised by the symmetric arrangements of points within the experimental region and a well chosen polynomial equation to represent the response surface over the entire simplex region. The polynomial has exactly as many parameters as there are number of points in the associated simplex lattice design. The response represents the property studied and is normally assumed to be a multi- varied function. In this study the response is the flexural strength.

Scheffe's modified polynomial equation using the restriction $\sum X_i = 1$, is represented as Eqn (2).

$$Y = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_{12} X_1 X_2 + \alpha_{13} X_1 X_3 + \alpha_{23} X_2 X_3 \quad (2)$$

The general form of Eqn (2) is

$$Y = \sum \alpha_i X_i + \sum \alpha_{ij} X_i X_j \quad (3)$$

where $1 \leq i \leq q, 1 \leq i < j \leq q$

q is the number of components of a mixture and i ranges from 1 to q .

X_i is the proportion of the i th component in the mixture.

α_i and α_{ij} are the coefficients.

The values of the unknown coefficients are determined using the following equations:

$$\alpha_i = y_i \quad (4)$$

$$\alpha_{ij} = 4y_{ij} - 2y_i - 2y_j \quad (5)$$

The pseudo components which represent the proportion of the components of the i th component in the mixture i.e. X_1, X_2, X_3 , were transformed to actual mix proportions (components) Z_1, Z_2, Z_3 using the following relationships and presented on Table 1.

$$X = BZ \quad (6)$$

$$Z = AX \quad (7)$$

where A = matrix whose elements are from the arbitrary mix proportions chosen when Eqn (7) is opened and solved mathematically.

B = the inverse of matrix A

Z = matrix of actual components

X = matrix of pseudo components obtained from the lattice.

Table 1: Pseudo and actual components for Scheffe's (3,2) lattice for soilcrete blocks

Pseudo Components				Response, Y	Actual Components		
No	X_1	X_2	X_3		Z_1	Z_2	Z_3
1	1	0	0	Y_1	0.8	1	8
2	0	1	0	Y_2	1	1	12.5
3	0	0	1	Y_3	1.28	1	16.67
4	0.5	0.5	0	Y_{12}	0.9	1	10.25
5	0.5	0	0.5	Y_{13}	1.04	1	12.335
6	0	0.5	0.5	Y_{23}	1.14	1	14.585
Control							
7	0.25	0.25	0.5	C_1	1.09	1	13.46
8	0.25	0.5	0.25	C_2	1.02	1	12.417
9	0.67	0.33	0	C_3	0.866	1	9.485
10	0	0.67	0.33	C_4	1.0924	1	13.8761
11	0.3	0.3	0.4	C_5	1.052	1	12.818
12	0.2	0.3	0.5	C_6	1.1	1	13.685

Legend:

X_1 = Water cement ratio

Z_1 = Actual water/cement ratio

X_2 = Fraction of cement

Z_2 = Actual cement quantity

X_3 = Fraction of laterite

Z_3 = Actual laterite quantity

Experimental method

The actual components as transformed from equation (7) and Table 1 were used to measure out the quantities water (Z_1), cement (Z_2), laterite (Z_3), for soilcrete blocks production in their respective ratios for the flexural strength test. Thirty-two blocks were tested for flexural strength using the hand operated flexural testing machine. The two point loading system was used. The load under which the specimen failed was recorded. The flexural strength was obtained from the following equation:

$$\sigma = WL/bh \quad (8)$$

where σ = the flexural strength

W = Maximum load

L = the distance between supporting rollers

b and h are the lateral dimensions of the specimen

Three blocks were tested for each point and the average taken as the flexural strength of the point.

VI. Result & Discussion/Experimental/Analysis/Implementation

The flexural strength results of the soilcrete blocks are presented on Table 2 and the replication variances of the test results are presented on (Table 3).

Table 2: Flexural strength test results of soilcrete blocks

Exp. No.	Mix ratios (w/c: cement: laterite)	Replicates	Mass (kg)	Density ρ (kg/m ³)	Average Density ρ (kg/m ³)	Failure Load (KN)	Flexural Strength σ (N/ mm ²)	Average σ (N/ mm ²)
1	0.8:1:8	A	18.2	1797.53		15.5	1.378	1.452
		B	18.2	1797.53	1774.49	17.5	1.556	
		C	17.5	1728.40		16.0	1.422	
2	1:1:12.5	A	15.2	1501.23		2.5	0.222	0.261
		B	16.0	1580.25	1547.32	3.5	0.311	
		C	15.8	1560.49		2.8	0.249	
3	1.28:1:1 6.67	A	17.3	1708.64		2.5	0.222	0.231
		B	15.3	1511.11	1609.88	2.3	0.204	
		C	16.3	1609.88		3.0	0.267	
4	0.9:1:10 .25	A	14.2	1402.47		3.5	0.311	0.279
		B	14.9	1471.60	1465.02	3.0	0.267	
		C	15.4	1520.99		2.9	0.258	
5	1.04:1:1 2.335	A	13.9	1372.84		2.1	0.187	0.267
		B	14.7	1451.85	1409.05	4.0	0.356	
		C	14.2	1402.47		2.9	0.258	
6	1.14:1:1 4.585	A	14.3	1412.35		2.2	0.196	0.228
		B	14.8	1461.73	1435.39	2.5	0.222	
		C	14.5	1432.10		3.0	0.267	
7	1.09:1:1 3.46	A	13.7	1353.09		2.5	0.222	0.196
		B	13.8	1362.96	1339.92	2.0	0.178	
		C	13.2	1303.70		2.1	0.187	
8	1.02:1:1 2.417	A	13.4	1323.46		2.1	0.187	0.208
		B	13.8	1362.96	1353.09	2.1	0.187	

		C	13.9	1372.84		2.8	0.249	
9	0.866:1: 9.485	A	15.8	1560.49		4.6	0.409	0.418
		B	12.8	1264.20	1389.30	5.0	0.444	
		C	13.6	1343.21		4.5	0.400	
10	1.0924:1 :13.8761	A	13.8	1362.96		4.0	0.356	0.305
		B	14.9	1471.60	1412.34	2.5	0.222	
		C	14.2	1402.47		3.8	0.338	
11	1.052:1: 12.818	A	15.4	1520.99		4.0	0.356	0.255
		B	14.0	1382.72	1455.15	2.5	0.222	
		C	14.8	1461.73		2.1	0.187	
12	1.1:1:13.685	A	12.5	1234.57		2.0	0.178	0.187
		B	14.6	1441.98	1326.75	2.0	0.178	
		C	13.2	1303.70		2.3	0.204	

Table 3: Flexural strength test result and replication variance (3-component mix)

Expt. No.	Replicates	Response Y_i (N/mm ²)	Response Symbol	$\sum Y_i$	Y	$\sum Y_i^2$	S_i^2
1	1A	1.378	Y_1	4.356	1.452	6.342	0.009
	1B	1.556					
	1C	1.422					
2	2A	0.222	Y_2	0.782	0.261	0.208	0.002
	2B	0.311					
	2C	0.249					
3	3A	0.222	Y_3	0.693	0.231	0.162	0.001
	3B	0.204					
	3C	0.267					
4	4A	0.311	Y_{12}	0.836	0.279	0.235	0.001
	4B	0.267					
	4C	0.258					

5	5A	0.187	Y ₁₃	0.801	0.267	0.228	0.007
	5B	0.356					
	5C	0.258					
6	6A	0.196	Y ₂₃	0.685	0.228	0.159	0.001
	6B	0.222					
	6C	0.267					
Control							
7	7A	0.222	C ₁	0.587	0.196	0.116	0.001
	7B	0.178					
	7C	0.187					
8	8A	0.187	C ₂	0.623	0.208	0.132	0.0013
	8B	0.187					
	8C	0.249					
9	9A	0.409	C ₃	1.253	0.418	0.524	0.001
	9B	0.444					
	9C	0.400					
10	10A	0.356	C ₄	0.916	0.305	0.290	0.005
	10B	0.222					
	10C	0.338					
11	11A	0.356	C ₅	0.765	0.255	0.211	0.008
	11B	0.222					
	11C	0.187					
12	12A	0.178	C ₆	0.560	0.187	0.105	0.000
	12B	0.178					
	12C	0.204					
						Σ	0.0373

The values of the mean of responses, Y and the variances of replicates S_i^2 presented in columns 6 and 8 of (Table 3) are gotten from the following Eqns (9) and (10):

$$y = \sum y/n \quad (9)$$

$$S_y^2 = [1/(n-1)]\{\sum y_i^2 - [1/n(\sum y_i)^2]\} \quad (10)$$

where $1 \leq i \leq n$

y_i = the responses

y = the mean of responses for each control point

n = control points

$n-1$ = degree of freedom

Considering all the design points, number of degrees of freedom,

$$V_e = \sum(N_{i-1}) \quad (11)$$

where $1 \leq i \leq 12$

$$V_e = 12 - 1 = 11$$

Replication variance,

$$S_y^2 = 1 / V_e \sum S_i^2 \quad (12)$$

$$= 0.0373 / 11 = 0.003391$$

Where S_i^2 is the variance at each point

$$\text{Replication error, } S_y = \sqrt{S_y^2} \quad (13)$$

$$= \sqrt{0.003391} = 0.058$$

This replication error value was used below to determine the t-statistics values for Scheffe's simplex model.

Determination of Scheffe's mathematical model for flexural strength of soilcrete blocks

From Eqns (4) and (5) and Table 3, the coefficients, α of the second degree polynomial are determined as follows:

$$\alpha_1 = 1.45, \quad \alpha_2 = 0.26, \quad \alpha_3 = 0.23$$

$$\alpha_{12} = 4(0.28) - 2(1.45) - 2(0.26) = -2.3$$

$$\alpha_{13} = 4(0.27) - 2(1.45) - 2(0.23) = -2.28$$

$$\alpha_{23} = 4(0.23) - 2(0.26) - 2(0.23) = -0.06$$

Substituting the values of these coefficients, α into Eqn (2) yields:

$$Y = 1.45X_1 + 0.26X_2 + 0.23X_3 - 2.3X_1X_2 - 2.28X_1X_3 - 0.06X_2X_3 \quad (14)$$

Eqn (14) is the Scheffe's mathematical model for optimisation of flexural strength of soilcrete block based on 28-day strength.

Test of the adequacy of the model

The model equation was tested for adequacy against the controlled experimental results. The statistical hypothesis for this mathematical model have been stated earlier

The student's t-test and fisher test statistics were used for this test. The expected values ($Y_{\text{predicted}}$) for the test control points were obtained by substituting the values of X_i from (Table 1) into the model equation i.e. 'equation (14)'. These values were compared with the experimental result (Y_{observed}) given in (Table 3).

Student's t-test

For this test, the parameters Δ_y , C and t are evaluated using the following equations respectively

$$\Delta_y = Y_{(\text{observed})} - Y_{(\text{predicted})} \quad (15)$$

$$C = (\sum a_i^2 + \sum a_{ij}^2) \quad (16)$$

$$t = \Delta_y \sqrt{n} / (S_y \sqrt{1 + C}) \quad (17)$$

where C is the estimated standard deviation or error,

t is the t-statistics,

n is the number of parallel observations at every point

S_y is the replication error

a_i and a_{ij} are coefficients while i and j are pure components

$$a_i = X_i(2X_i - 1)$$

$$a_{ij} = 4X_iX_j$$

$$Y_{\text{obs}} = Y_{(\text{observed})} = \text{Experimental results}$$

$$Y_{\text{pre}} = Y_{(\text{predicted})} = \text{Predicted results}$$

Table 4: T-statistics test computations for Scheffe's flexural strength model

N	CN	i	j	a_i	a_{ij}	a_i^2	a_{ij}^2	ϵ	$Y_{(\text{observed})}$	$Y_{(\text{predicted})}$	Δ_y	t
1	C ₁	1	2	-0.125	0.25	0.01562	0.0625	0.6094	0.196	0.106	0.09	2.12
		1	3	-0.125	0.5	0.01562	0.25					
		2	3	-0.125	0.5	0.01562	0.25					
		3	-	0	-	0	-					
					Σ	0.04686	0.5625					
Similarly												
2		-	-	-	-	-	-	0.6094	0.208	0.113	0.095	2.24
3		-	-	-	-	-	-	0.899	0.418	0.549	0.131	2.84

4	-	-	-	-	-	-	0.8476	0.305	0.237	0.068	1.49
5	-	-	-	-	-	-	0.640	0.25	0.12	0.13	3.03
6	-	-	-	-	-	-	0.6208	0.187	0.108	0.079	1.85

T-value from table

For a significant level, $\alpha = 0.05$, $t_{\alpha/2}(v_e) = t_{0.05/2}(5) = t_{0.01}(5) = 3.365$ (see standard t-table given in Appendix II).

This value is greater than any of the t-values obtained by calculation (as shown in Table 4). Therefore, we accept the Null hypothesis. Hence the model equation is adequate.

Fisher Test

For this test, the parameter y , is evaluated using the following equation:

$$y = \sum Y/n \quad (18)$$

where Y is the response and n the number of responses.

Using variance, $S^2 = [1/(n-1)][\sum (Y-y)^2]$ and $y = \sum Y/n$ for $1 \leq i \leq n$ (19)

The computation of the fisher test statistics is presented in Table 5.

Table 5: F-statistics test computations for Scheffe's flexural strength model

Response Symbol	$Y_{(observed)}$	$Y_{(predicted)}$	$Y_{(obs)} - Y_{(obs)}$	$Y_{(pre)} - Y_{(pre)}$	$(Y_{(obs)} - Y_{(obs)})^2$	$(Y_{(pre)} - Y_{(pre)})^2$
C_1	0.196	0.106	-0.06467	-0.0995	0.004182	0.0099
C_2	0.208	0.113	-0.05267	-0.0925	0.002774	0.008556
C_3	0.418	0.549	0.157333	0.3435	0.024754	0.117992
C_4	0.305	0.237	0.044333	0.0315	0.001965	0.000992
C_5	0.25	0.12	-0.01067	-0.0855	0.000114	0.00731
C_6	0.187	0.108	-0.07367	-0.0975	0.005427	0.009506
Σ	1.564	1.233			0.039215	0.154258
	$y_{(obs)}=0.260667$	$y_{(pre)}=0.2055$				

Legend: $y = \sum Y/n$

where Y is the response and n the number of responses.

Using Eqn (19), $S^2_{(obs)}$ and $S^2_{(pre)}$ are calculated as follows:

$$S^2_{(obs)} = 0.039215/5 = 0.007843 \text{ and } S^2_{(pre)} = 0.154258/5 = 0.0308516$$

The fisher test statistics is given by:

$$F = S_1^2 / S_2^2 \quad (20)$$

where S_1^2 is the larger of the two variances.

Hence $S_1^2 = 0.0308516$ and $S_2^2 = 0.007843$

Therefore, $F = 0.0308516 / 0.007843 = 3.9$

From Fisher table (Appendix JJ), $F_{0.95}(5,5) = 5.1$ which is higher than the calculated F-value. Hence the regression equation is adequate.

VII. Recommendations

The use of Soilcrete blocks is strongly recommended in areas where late rite is available.

VIII. Conclusion

1. Readily available and affordable late rite has been used successfully to produce Soilcrete blocks. With this, there will be reduced dependence on river sand which is costly, scarce and unavailable in some places.
2. A model for predicting flexural strength of the blocks has been developed using Scheffe's simplex lattice method.
3. Adequacy tests were carried out on the model equation using student's t-test and the fisher test. They proved the model equation to be adequate.

4. The optimum value of flexural strength obtained from the model is 1.452N/mm^2 .
5. With the fundamental model, the flexural strength of the soilcrete blocks can be determined if the mix ratios are stipulated. In the reverse order, the mix ratios can be predicted if the value of the flexural strength is defined.

IX. Scope for Further Research

The relationship between the flexural strength and other properties of soilcrete blocks should be established

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