International Journal of Computer Sciences and Engineering Open Access
Research Paper Vol.-6, Issue-10, Oct 2018 E-ISSN: 2347-2693

AVAILABILITY ANALYSIS OF TWO SYSTEM WITH AND WITHOUT PREVENTIVE MAINTENANCE

Praveen Gupta[1]*, Pooja Vinodiya[2]

[1,2]School of Studies in Statistics, Vikram University, Ujjain (M.P.) – 456010, India

Corresponding Author: poojavinodia10@gmail.com

Available online at: www.ijcseonline.org

Accepted: 15/Oct/2018, Published: 31/Oct/2018

***Abstract*:** This paper deals with two systems have a single unit with two dissimilar components. The system remains operative even if a single component is in operative mode. The failure of one component creates change in life time parameter of other component. Both components can be replaced with a similar component. After replacement of each component, the system is as good as new. In second system Preventive maintenance (i.e. inspection, minor repair etc) is provide to system when system is in the state S_0 where both components are normal mode.

***Keywords*:** Reliability**,** Availability, Busy Period Analysis, Mean Sojourn Time, Transition Probabilities and Preventive Maintenance.

I. INTRODUCTION

In reliability field several authors have attracted to complex redundant system. Some of them obtain the availability of a system with two sub system taking constant failure rate and general repair time distribution for each system. Preventive maintenance is maintenance that is regularly performed on a piece of equipment to lessen the likelihood of it failing. It is performed while the equipment is still working so that it does not break down unexpectedly. It is planned maintenance that ensures any required resources are available.

The maintenance is scheduled based on a time or usage trigger. A typical example of an asset with a time-based preventative maintenance program schedule is an air-conditioner which is serviced every year, before summer. Keeping this concept in view, the purpose of the present paper is to compare the two systems one is without preventive maintenance and second is with preventive maintenance.

II. STATES AND SYMBOLS OF SYSTEMS

These are the common symbols for both the systems

θ_1 = Failure rate of component A.

θ_2 **=** Failure rate of component B.

$\theta_1^{'}$ = Failure of A when B has already failure.

$\theta_2^{'}$ = Failure of B when A has already failure.

α_1 = Replacement of A.

α_2 = Replacement of B.

α = Replacement of A and B.

g(t) = Pdf of time for taking a limit into preventive maintenance.

$g(t) = \lambda e^{-\lambda t}$

h(t) = pdf of preventive maintenance time

$h(t) = \theta e^{-\theta t}$.

A_0, B_0 = Component A and B in operative mode.

A_0, B_F = Component A is in operative mode and B in Failure mode.

A_F, B_0 = Component **B** is in operative mode and A in Failure mode.

A_F, B_F = Component **A** and B in Failure mode.

A_{pm}, B_{pm} = Component A and B are in Preventive Maintenance.

FOR SYSTEM-1

UP STATES:

$S_0 = (A_0, B_0)$; $S_1 = (A_0, B_F)$

$S_2 = (A_F, B_0)$

DOWN STATE:

$S_3 = (A_F, B_F)$

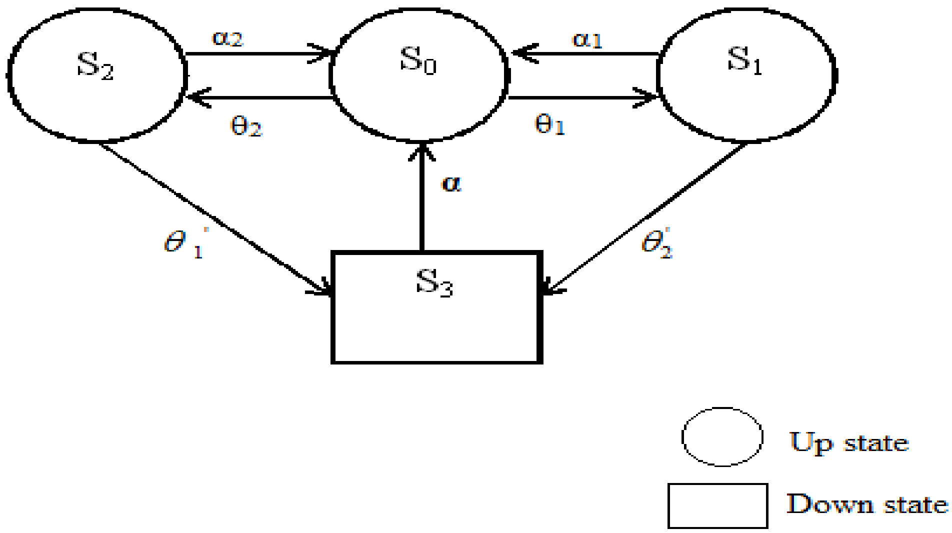


Figure 1: TRANSITION DIAGRAM FOR SYSTEM-1

III. STEADY STATE TRANSITION PROBABILITIES

Let p_{ij} denotes the steady state probability that the system transits from state S_i to S_j (i, j= 1,2,……..9)

$$p_{ij} = \lim_{t\to\infty} Q_{ij}(t) \; and \; p_{ij}^{k} = \lim_{t\to\infty} Q_{ij}^{k}(t)$$

Therefore,

$$P_{01} = \frac{\theta_1}{\theta_2 + \theta_1} \quad , \qquad P_{02} = \frac{\theta_2}{\theta_2 + \theta_1}$$

$$P_{10} = \frac{\alpha_1}{\alpha_1 + \theta_2^{'}} \quad , \qquad P_{13} = \frac{\theta_2^{'}}{\alpha_1 + \theta_2^{'}}$$

$$P_{20} = \frac{\alpha_2}{\alpha_2 + \theta_1^{'}} \quad , \qquad P_{23} = \frac{\theta_1^{1}}{\alpha_2 + \theta_1^{'}}$$

$$P_{30} = 1$$

IV. MEAN SOJOURN TIME

Mean time of stay in state S_i is called the mean sojourn time in state S_i and it is denoted by Ψ_i. If T_i is the sojourn time in state S_i , then the mean sojourn time in state S_i is given by :

$$\psi_i = \int P(T_i. > t)\, dt$$

$$\Psi_0 = \frac{1}{\theta_1 + \theta_2} \quad , \qquad \Psi_1 = \frac{1}{\alpha_1 + \theta_2^{''}}$$

$$\Psi_2 = \frac{1}{\alpha_2 + \theta_1^{'}} \qquad \Psi_3 = \frac{1}{\alpha}$$

V. RELIABILITY OF THE SYSTEM AND MTSF

Let R_i (t) denotes the probability that the system is operable up to time t when it starts from state S_i. In order to obtain R_0 (t), we consider the following two contingencies.

1. System remains up in state S_0 without making any transition to any other state up to time t. The probability of this contingency is

$$e^{-(\theta_1+\alpha_1)t} = Z_0(t)$$

2. System enters to the state S_1 from S_o during (u, u+du), $u \le t$ and then starting from S_1, it remains up continuously during remaining time (t-u), the probability of this contingency is

$$\int_0^t q_{01}(u)\,du R_1(t-u) = q_{01} © R_1(t)$$

Then

$$R_0(t) = Z_0(t) + q_{01} © R_1(t) + q_{02} © R_2(t) \qquad ..(1)$$

1.

$$R_1(t) = Z_1(t) + q_{10} © R_0(t)$$

$$R_2(t) = Z_2(t) + q_{20} © R_0(t) \qquad ..(2\text{-}3)$$

(2-3)

Taking Laplace transforms of the above equations. For brevity, We have omitted the argument 's' from q^*_{ij}(s), Z^*_i(s) and R^*_i(s). Solving the above matrix equation for R^*_0(s), we get

$$R^*_0(s) = \frac{N_1(s)}{D_1(s)}$$

Where

$$N_1(s) = Z^*_0 + q^*_{01} Z^*_1 + q^*_{02} Z^*_2$$

$$D_1(s) = 1 - \{ q^*_{01} q^*_{10} + q^*_{02} q^*_{20} \}$$

The mean time to system failure (MTSF) can be obtained by using the formula,

$$E(T_0) = \int R_0 (t)\, dt$$

$$= \lim_{s \to 0} R^*_0 (s)$$

$$= \frac{N_1(0)}{D_1(0)} \quad (4)$$

(4)

To determine the $N_1(0)$ and $D_1(0)$, we first obtain $Z^*_i(0)$ by using the following result

$$\lim_{s \to 0} Z^*_i(s) = \int Z_i (t)\, dt$$

Therefore,

$$Z^*_0(0) = \psi_0 \text{ and } Z^*_i(0) = \psi_i \quad (i = 1, 2, 3)$$

We can get the MTSF of the system by putting these values of $N_1(0)$ and $D_1(0)$ in (4).

VI. AVAILABILITY ANALYSIS

1. When unit is in normal (N) mode and operative: Let A^i_N (t) be the probability that the system is up, due to a unit in N-mode at epoch t, when it initially start from state S_i € E

$$A^N_0(t) = Z_0(t) + q_{01}(t) © A^N_1(t)$$

$$A^N_1(t) = Z_1(t) + q_{10} © A^N_0(t) + q_{13} © A^N_3(t)\; A^N_2(t) =$$

$$Z_2(t) + q_{23}(t) © A^N_3(t) + q_{20}(t) © A^N_0(t)$$

$$A^N_3(t) = q_{30}(t) © A^N_0(t)$$

After taking Laplace Transformation and solving these equation we get

$$A^{N*}_0(s) = \frac{N_2(s)}{D_2(s)} \quad (5)$$

5.

Where.

$$N_2(s) = Z^*_0 + q^*_{01} Z^*_1 + q^*_{02} Z^*_2$$

$$D_1(s) = 1 - \{ q^*_{01} q^*_{10} + q^*_{13} q^*_{30}) + q^*_{02}(q^*_{23} q^*_{30} + q^*_{20})$$

Therefore, the steady state availability of the system, due to a unit in N-mode and operative, is given by

$$A^N_0 = s\frac{N_2(s)}{D_2(s)} \quad (6)$$

6.

Since $D_2(0)$ =0, therefore by L- Hospital law from 6 we have

$$A^N_0 = \frac{N_2(0)}{D_2(0)} \quad (7)$$

7.

FOR SYSTEM-2
UP STATES:

$S_0 = (A_0, B_0)$; $S_1 = (A_0, B_F)$

$S_2 = (A_F, B_0)$: $S_3 = (A_{pm}, B_{pm})$

DOWN STATE:

$S_4 = (A_F, B_F)$

TRANSITION DIAGRAM:

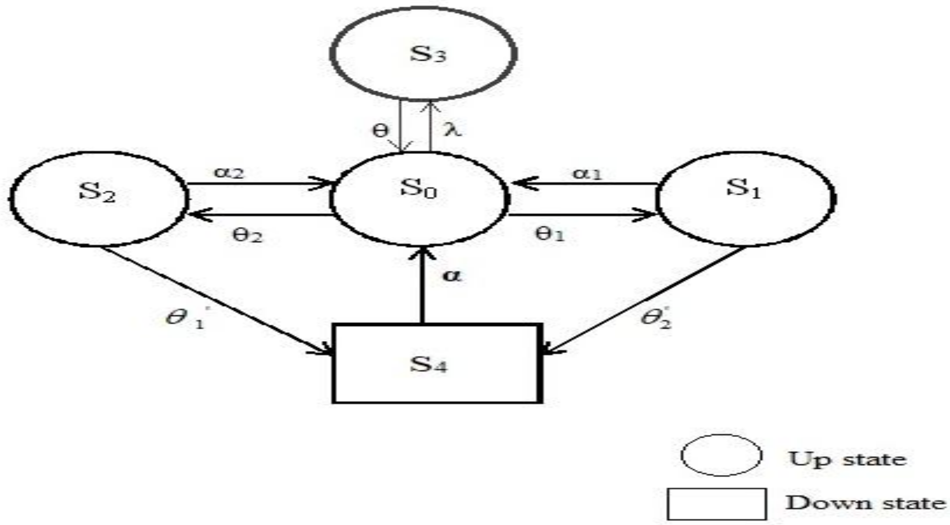


Figure- 2 TRANSITIOM DIAGRAM FOR SYSTEM-2

VII. STEADY STATE TRANSITION PROBABILITIES

Let p_{ij} denotes the steady state probability that the system transits from state S_i to S_j (i, j= 1,2,……..9)

$$p_{ij} = \lim_{t\to\infty} Q_{ij}(t) \; and \; p_{ij}^{k} = \lim_{t\to\infty} Q_{ij}^{k}(t)$$

Therefore,

$$P_{01} = \frac{\theta_1}{\theta_2 + \theta_1 + \lambda}, \qquad P_{02} = \frac{\theta_2}{\theta_2 + \theta_1 + \lambda}$$

$$P_{03} = \frac{\lambda}{\theta_2 + \theta_1 + \lambda}, \qquad P_{10} = \frac{\alpha_1}{\alpha_1 + \theta_2^{'}}$$

$$P_{14} = \frac{\theta_2^{1}}{\alpha_1 + \theta_2^{'}}, \qquad P_{20} = \frac{\alpha_2}{\alpha_2 + \theta_1^{'}}$$

$$P_{24} = \frac{\theta_1^{1}}{\alpha_2 + \theta_1^{'}}, \qquad P_{30} = 1$$

$$P_{40} = 1.$$

VIII. MEAN SOJOURN TIME

Mean time of stay in state S_i is called the mean sojourn time in state S_i and it is denoted by Ψ_i. If T_i is the sojourn time in state S_i , then the mean sojourn time in state S_i is given by :

$$\psi_i = \int P(T_i. > t)\, dt$$

$$\Psi_0 = \frac{1}{\theta_1 + \theta_2 + \lambda}, \quad \Psi_1 = \frac{1}{\alpha_1 + \theta_2^{''}} \quad \Psi_2 = \frac{1}{\alpha_2 + \theta_1^{'}}$$

$$\Psi_3 = \frac{1}{\theta} \qquad \Psi_4 = \frac{1}{\alpha}$$

IX. RELIABILITY OF THE SYSTEM AND MTSF

Let R_i (t) denotes the probability that the system is operable up to time t when it starts from state S_i. In order to obtain R_0 (t), we consider the following two contingencies.

1. System remains up in state S_0 without making any transition to any other state up to time t. The probability of this contingency is

$$e^{-(\theta_1 + \alpha_1)t} = Z_0(t)$$

2. System enters to the state S_1 from S_o during (u, u+du), u $\leq$t and then starting from S_1, it remains up continuously during remaining time (t-u), the probability of this contingency is

$$\int_0^t q_{01}(u) du R_1(t-u) = q_{01} © R_1(t)$$

Then

$$R_0(t) = Z_0(t) + q_{01} © R_1(t) + q_{02} © R_2(t) + q_{03} © R_3(t) \quad (8)$$

$$R_1(t) = Z_1(t) + q_{10} © R_0(t) \quad (9)$$

$$R_2(t) = Z_2(t) + q_{20} © R_0(t) \quad (10)$$

$R_3(t) = Z_3(t) + q_{30} © R_0(t)$ (11)

Taking Laplace transforms of the above equations. For brevity, We have omitted the argument 's' from q^*_{ij}(s), Z^*_i(s) and R^*_i(s). Solving the above matrix equation for R^*_0(s), we get

$$R^*_0(s) = \frac{N_1'(s)}{D_1'(s)}$$

Where

$$N_1'(s) = Z^*_0 + q^*_{01} Z^*_1 + q^*_{02} Z^*_2 + Z^*_3 q^*_{30}$$

$$D_1'(s) = 1 - \{ q^*_{01} q^*_{10} + q^*_{02} q^*_{20} + q^*_{30} q^*_{03} \}$$

The mean time to system failure (MTSF) can be obtained by using the formula,

$$E(T_0) = \int R_0(t)\,dt = \lim_{s\to 0} R^*_0(s) = \frac{N_1'(0)}{D_1'(0)} \quad \ldots(12)$$

To determine the $N_1'(0)$ and $D_1'(0)$, we first obtain $Z^*_i(0)$ by using the following result

$$\lim_{s\to 0} Z^*_i(s) = \int Z_i(t)\,dt$$

Therefore,

$$Z^*_0(0) = \psi_0 \text{ and } Z^*_i(0) = \psi_i \quad (i = 1, 2, 3, 4)$$

We can get the MTSF of the system by putting these values of $N_1(0)$ and $D_1(0)$ in (12).

X. AVAILABILITY ANALYSIS

1. When unit is in normal (N) mode and operative

Let $A^i_N(t)$ be the probability that the system is up, due to a unit in N-mode at epoch t, when it initially start from state S_i € E

11

$$A^N_0(t) = Z_0(t) + q_{01}(t) © A^N_1(t) + q_{02}(t) © A^N_2(t)$$

$$A^N_1(t) = Z_1(t) + q_{10} © A^N_0(t) + q_{13} © A^N_3(t)$$

$$A^N_2(t) = Z_2(t) + q_{23}(t) © A^N_3(t) + q_{20}(t) © A^N_0(t)$$

$$A^N_3(t) = Z_3(t) + q_{30}(t) © A^N_0(t)$$

$$A^N_4(t) = q_{40}(t) © A^N_0(t)$$

After taking Laplace Transformation and solving these equation we get

$$A^{N*}_0(s) = \frac{N_2'(s)}{D_2'(s)} \quad (13)$$

Where.

$$N_2'(s) = Z^*_0 + q^*_{01} Z^*_1 + q^*_{02} Z^*_2 + q^*_{03} Z^*_3$$

$$D_2'(s) = 1 - \{ q^*_{01} (q^*_{10} + q^*_{14} q^*_{40}) + q^*_{02}(q^*_{24} q^*_{40} + q^*_{20}) + q^*_{30} q^*_{03}$$

Therefore, the steady state availability of the system, due to a unit in N-mode and operative, is given by

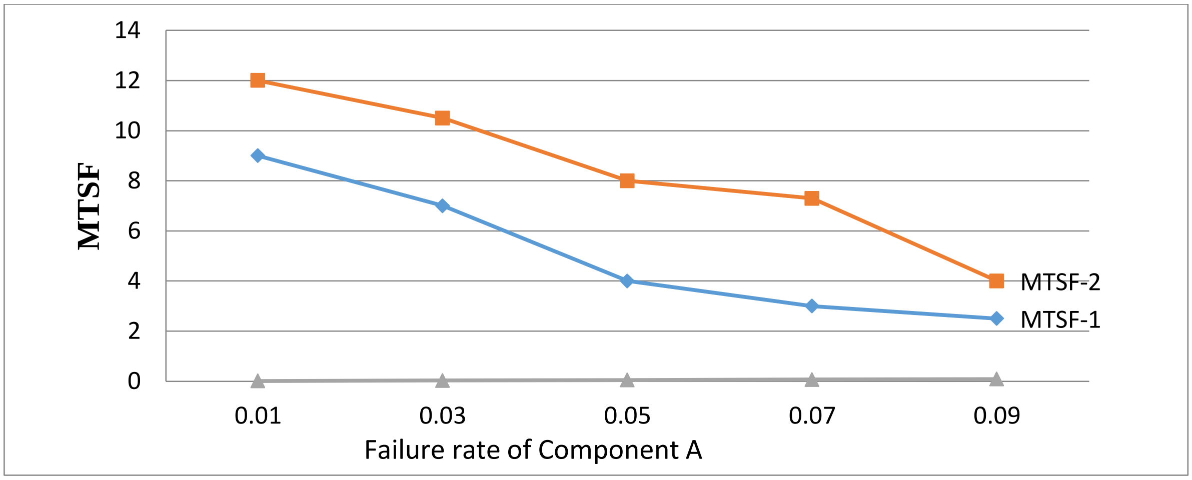
$$A^N_0 = s\,\frac{N_2'(s)}{D_2'(s)} \quad (14)$$

Since $D_2(0)$ =0, therefore by L- Hospital law from 6 we have

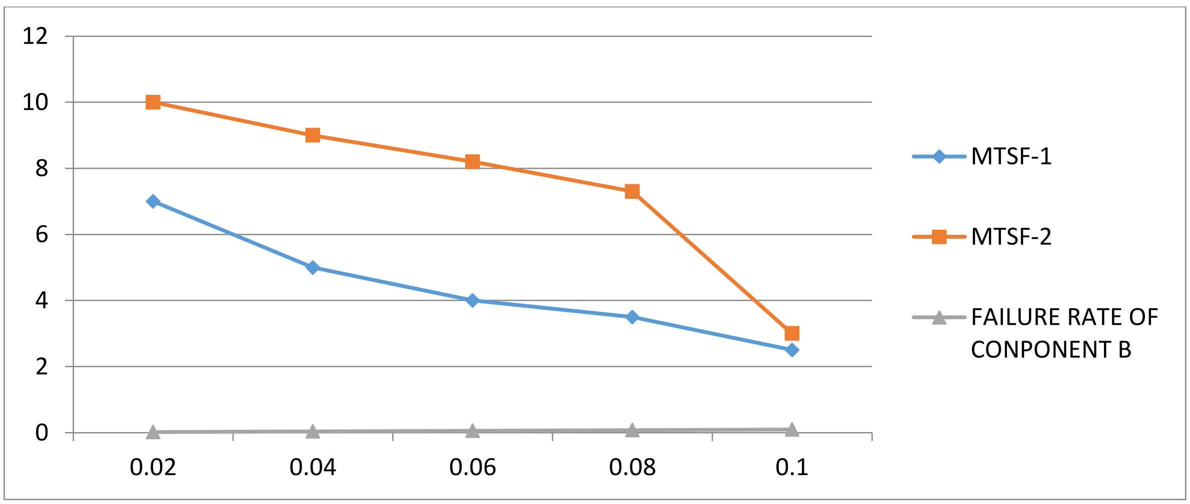
$$A^N_0 = \frac{N_2'(0)}{D_2'(0)} \quad (15)$$

GRAPHICAL STUDY:

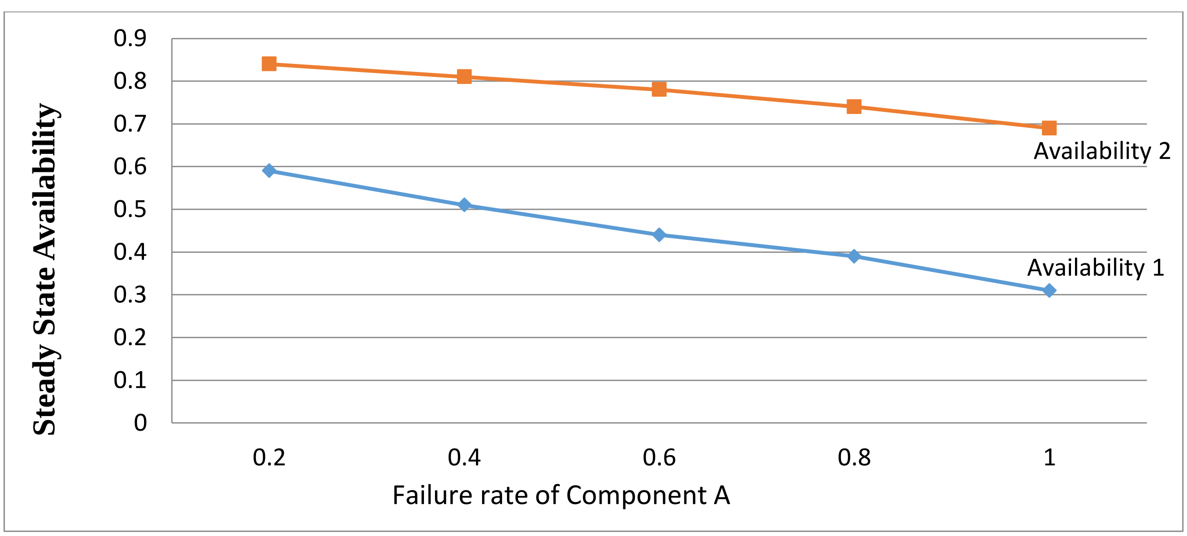
For a more concrete study of the system behaviour, we plot curves for MTSF and Availability for the different values of $\theta_1$andθ_2 in System-1 and System-2, respectively keeping the other parameter fixed.



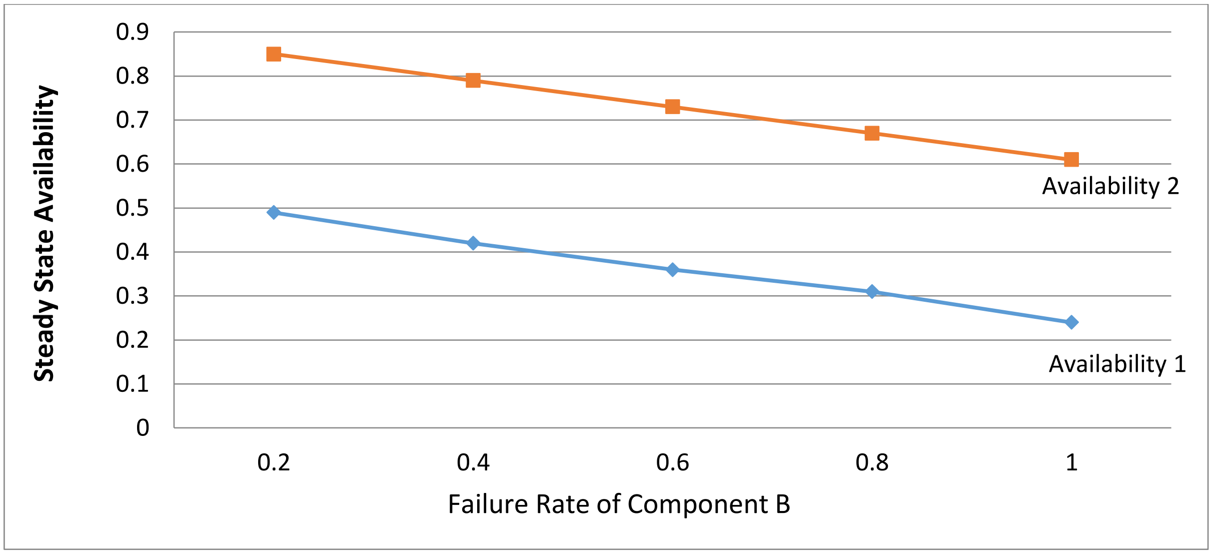
Behaviour of MTSF against the failure rate of component A



Behaviour of MTSF against the failure rate of component B



Steady State Availability against the failure rate of component A



Steady State Availability against the failure rate of component B

XI. CONCLUSION

After Graphical study we can see that the MTSF and availability of system-2 is better than system-1 because in system-2 we provide it a preventive maintenance which is responsible for the betterment of system-2.

REFERENCES

[1]. Goel, L.R., Srivastava P. and Gupta R, "Two unit cold standby system with correlated failures and repairs". International journal of system science,Vol 23,Issue3, 379-391,1992.
[2]. Goel, L.R. and Gupta Praveen, "Stocastic behaviour of a two unit (dissimilar) hot standby system with three modes", Microelectron relib. vol. 23 No 6 pp 1035-1040, 1983.
[3]. Goel, L.R. and P. Gupta, "Stochastic analysis of a two-unit parallel system with partial and catastrophic failures and preventive maintenance", Microelectron relib. vol. 24, pp. 881-883, 1984.
[4]. Gupta, R. and G. Varshney, "A two-identical unit parallel system with geometric failure and repair time distributions",Journal of Cobinatories, Information and System Science, Vol. 32, issue. 2, pp.127-136,1984.
[5]. Goel, L.R., Srivastava P. and Gupta R. "Comparison of reliability characteristics of two system with bivariate exponential lifetime", International journal of system science, Vol 23, issue 3, pp.135-144,1992.

Author Profile

Ms.Pooja Vinodiya pursed Bachelor of Science from G.D.C. College, Ujjain in 2010 and Master of Science from SOS in Statistics, Vikram University (Gold Medalist) in year 2012. She is currently pursuing Ph.D. and awarded JRF with Rajeev Gandhi National Fellowship, UGC (New Delhi) in year 2014. She is published two research papers in reputed international journals and conference including ISSAC-2016 at Aligarh Muslim University and National Conference at Savitribai Phule Pune University. Her main research work focus on Reliabiliy Analysis, Availability Analysis, Prevantive Maintenance of system. She has five years reasearch expierence.