JCSE International Journal of Computer Sciences and Engineering Open Access
Research Paper Vol.-7, Issue-6, June 2019 E-ISSN: 2347-2693

Fuzzy Labeling on Cycle Related Graph

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DOI: https://doi.org/10.26438/ijcse/v7i6.11921194 | Available online at: www.ijcseonline.org

Accepted: 22/Jun/2019, Published: 30/Jun/2019

Abstract— In this paper the concept of fuzzy labeling of fan graph has been introduced, the generalized formula for *n*-dimensional fan of the vertices and edges and the new algorithm of fuzzy labelling of fan graph have been discussed. Every membership function of fuzzy labeling of a fan graph must be distinct.

Keywords— fuzzy path , fuzzy graph, fuzzy labeling graph, fan graph

I. INTRODUCTION

In 1965, Zadeh introduced the concept of uncertainty by fuzzy set [1]. A fuzzy set is defined mathematically by assigning each possible individual in the universe of discourse a value, representing its grade of membership, which corresponds to the degree, to which that individual is similar or compatible with the concept represented by the fuzzy set.

The fuzzy graph was introduced by Rosenfeld using fuzzy relation which represents the relationship between the objects by previously indicating the level of the relationship between the objects of the function sets [2]. Fuzzy graphs have many applications in modelling real time systems where the level of information inherent in the system varies with different levels of precision [3].The basic concept of fuzzy graph and fuzzy labelling graph were discussed.

In this paper section 1 deal with fuzzy labeling related results and section 2 deals with fuzzy labelling of fan graph and. Section 3 contains the results of fuzzy labelling of fan graph, algorithm and verification.

II. PRELIMINARIES

Let U and V be two sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of $U \times V$. A fuzzy graph $G = (V, \sigma, \mu)$ of a nonempty set V together with a pair of function $\sigma : V \rightarrow [0,1]$ and $\mu : E \rightarrow [0,1]$,it satisfy $\mu(v,u) \leq \sigma(v) \wedge \sigma(u)$ for all $v, u \in V$. A path P in a fuzzy graph is a sequence of distinct nodes $v_1, v_2 \dots v_n$ such that $\mu(v_1, v_{i+1}) > 0$; $1 \leq i \leq n$; here $n \geq 1$ is called the length of the path P. The consecutive pairs (v_i, v_{i+1}) are called the edges of the path.

A path P is called a cycle if $v_1 = v_n$ and $n \geq 3$. The strength of a path P is defined as the weight of the weakest arc of the path. An arc of a fuzzy graph is called strong if its weight is at least as great as the strength of the connectedness of its end nodes. A graph $G = (V, \sigma, \mu)$ is said to be a fuzzy labelling graph, if $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$ is bijective such that the membership value of edges and vertices are distinct and $\mu(u,v) < \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ [4].

An elegant definition of a metric in a fuzzy graph has been given by Rosenfeld [5]. If ρ is the path consisting of the vertices $x_1, x_2 \dots x_n$ in a fuzzy graph $G = (\tau, \gamma)$, the μ- length fuzzy graph is define by $l(\rho)$ where $l(\rho) = \sum_{i=1}^{n} \mu(x_{i-1}, x_i)^{-1}$, the μ- length of all paths joining X and Y. The μ- distance $\delta(u,v)$ is the smallest μ -length of any $u - v$ path and δ is metric. Suppose $G = (\sigma, \mu)$ is a fuzzy graph with V as the set of vertices. The eccentricity $e(v)$ of a vertex $v \in V$ is defined to be the maximum of all the μ- distance $\delta(v, u)$ for all u in V. The radius of a connected fuzzy graph is the minimum of all eccentricities of the vertices of the fuzzy graph. An eccentric node v, is a node v^* such that $e(v) = \delta(v, v^*)$. A node v is called a diametrical node if $e(v) = diam(G)$ [3] [6]. If G* is a cycle then the fuzzy labeling cycle G_ω contain exactly only one weakest arc [6].

Definition: A cycle graph G∗ is said to be a fuzzy labeling cycle graph if it has fuzzy labelling [6].
Proposition: Let G_ω be a fuzzy labeling cycle such that G* is a cycle, then it has (n-1) bridges

Proposition: Let $G = (\sigma_\omega , \mu_\omega)$ be a fuzzy graph such that G* is a cycle. Then a node is a fuzzy cut node of G if and only if it is a common node of two fuzzy bridges [7]

Proposition: If *G** is a cycle with fuzzy labeling then it has (*n*-2) cut nodes [6,7]

III. METHODOLOGY

1.Fuzzy graphs

A fuzzy graph G = (V, σ, μ) is a triple consisting of a nonempty set V together with a pair of functions σ : V → [0, 1] and μ : E → [0, 1] such that for all x, y∈ V, μ(x , y) ≤ σ(x) Λ σ(y).

2.Fuzzy labeling

A graph G = (σ, μ) is said to be a fuzzy labeling graph, if σ: V → [0, 1] and μ: V×V → [0, 1] are bijective such that the membership value of edges and vertices are distinct and μ (u, v) < σ (u) Λ σ (v) for all u, v ϵ V.

Proposition: If *G** is a cycle with fuzzy labeling then, the graph has exactly two end nodes.

Proposition: If G_ω is a fuzzy labeling cycle graph, then every bridge is strong and vice versa.

Proposition: If G_ω is a connected fuzzy labeling graph then there exists a strong path between any pair of nodes.

Proposition: Every fuzzy labeling graph has atleast one weakest arc.

Proposition: For any fuzzy labeling graph G_ω , $\delta(G_\omega)$ is a fuzzy end node of G ω such that the number of arcs incident on, $\delta(G_\omega)$ is at least two.

Proposition: Every fuzzy labeling graph has at least one end nodes.[1].

III. RESULTS AND DISCUSSION

1. Fuzzy labeling of Fan graph

A fan graph denoted by F_n is the path P_{n-1} plus and extra vertex connected to all vertices of the path P_{n-1} [7]. A fuzzy fan graph $G_f = (V, \tau, \gamma)$ of a nonempty set V together with a pair of functions $\tau : V \rightarrow [0,1]$ and $\gamma : E \rightarrow [0,1]$. It is denoted by G_f and satisfies the condition

$$\gamma(v_k, v_l) \leq \tau(v_k) \wedge \tau(v_l) \text{ for all } v_i, v_j \in V$$

Theorem 1: Every Fan graph admits fuzzy labelling of a graph.

Proof: Let G be Fan graph, it's vertices and edges are denoted by F_n .

A fuzzy fan graph $G(V, \tau, \gamma)$ is a non-empty set V together with a pair of function $\tau : V \rightarrow [0,1]$ and $\gamma : E \rightarrow [0,1]$. it is denoted by G_f and for all $v_k, v_l \in V$,

$$\gamma(v_k, v_l) \leq \tau(v_k) \wedge \tau(v_l) \rightarrow (1.1)$$

A fuzzy labeling of a graph $G_f = (V, \tau, \gamma)$ is a nonempty set V together with a pair of functions $\tau : V \rightarrow [0,1]$ and $\gamma : E \rightarrow [0,1]$. It satisfies the condition

$$\gamma(v_k, v_l) < \tau(v_k) \wedge \tau(v_l) \rightarrow (1.2) \text{ for all } v_k, v_l \in V$$

Let us consider the *n*- dimension of fan graph,and the path P_{n-1} plus and extra vertex connected to all vertices of the path P_{n-1}, $n \geq 3$

Define the membership function of the vertices of a fan graph is

$$\tau(v_i) = \frac{2n+i-3}{10^{n-2}} \text{ where } i = 1,2,3\ldots.n$$

Case (i)

Let us consider the set of vertices of the path P_{n-1} in a graph G_f, is $v_1, v_2, v_3, v_4 \ldots v_i$ then every pair of the vertices of the edges mapping

$$\gamma(e_i): v_i \times v_{i+1} \rightarrow \frac{i}{10^{n-2}} \text{ where } i = 1,2,3\ldots.n\text{-}1$$

Case (ii)

Let v_n be the vertex adjacent with every pair vertex of path P_{n-1}, the mapping of the edges defined by

$$\gamma(e_{n+i-2}): v_n \times v_i \rightarrow \frac{n+i-2}{10^{n-2}} \text{ where } i = 1,2,3\ldots k$$

Here every membership function of vertices $\tau(v_1), \tau(v_2), \tau(v_3) \ldots \tau(v_n)$ and the edges $\gamma(v_k, v_l)$ are distinct, and it satisfies the equation 1.1 and 1.2

Hence fan graph admits Fuzzy labelling of graph.

Algorithm of fuzzy labelling on fan graph:

Step:1 Fix *n*, the dimension of a fan graph

Step:2 Fix the membership value to edges of path P_{n-1} by

$$e_i: v_i \times v_{i+1} \rightarrow \frac{i}{10^{n-2}} \text{ where } i = 1,2,3\ldots.$$

Step:3 Fix the membership value of remaining edges whose vertices adjacent with v_n.by

$$e_{n+i-2}: v_n \times v_i \rightarrow \frac{n+i-2}{10^{n-2}} \text{ where } i = 1,2,3\ldots.$$

Step:4 After giving membership function to edges, continue to give membership value vertices by

$$\tau(v_i) = \frac{2n+i-3}{10^{n-2}} \text{ where } i = 1,2,3\ldots.$$

Step:5 step 1,step 2,step 3 must satisfies $\gamma(v_k, v_l) < \tau(v_k) \wedge \tau(v_l) \rightarrow (1.2)$ for all $v_k, v_l \in V$ otherwise repeat step1.

Verification:

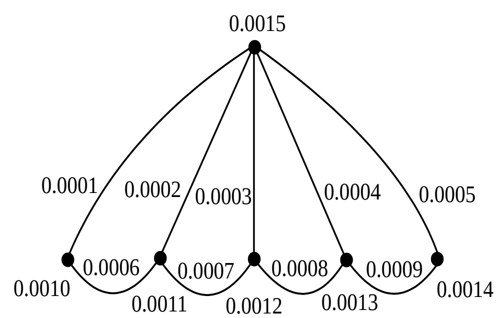

Figure 1.a: Fuzzy labelling of fan graph

Step:1 fix $n = 6$

Step:2 In any consecutive path

$i = 1$ then $e_1: v_1 \times v_2 \rightarrow \frac{1}{10^{6-2}}$ which implies membership of $e_1$is 0.0001

i = 2 then $\gamma(e_2) = 0.0002$
i = 3 then $\gamma(e_3) = 0.0003$
i = 4 then $\gamma(e_4) = 0.0004$
Step:3 v_6 adjacent with v_1, v_2, v_3, v_4, v_5.

i = 5 then e_{6+1-2}: $v_6 \times v_1 \rightarrow \frac{6+1-2}{10^{6-2}}$ (i,e) $\gamma(e_5) = 0.0005$
i = 6 then $\gamma(e_6) = 0.0006$
i = 7 then $\gamma(e_7) = 0.0007$
i = 8 then $\gamma(e_8) = 0.0008$
i = 9 then $\gamma(e_9) = 0.0009$
Step:4 There are 6 vertices of fan graph F_6 v_1, v_2, v_3, v_4, v_5, v_6 Membership function of the vertices are,
$\tau(v_1) = \frac{2(6)+1-3}{10^{6-2}} = 0.0010$
$\tau(v_2) = 0.0011$
$\tau(v_3) = 0.0012$
$\tau(v_4) = 0.0013$
$\tau(v_5) = 0.0014$
$\tau(v_6) = 0.0015$

IV. CONCLUSION

In this paper, the concept of fuzzy labeling has been derived for fan graphs. Fuzzy labeling for cycle related have been discussed. We further extend study on interconnection networks and cycle free graphs.

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