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Study of Topological Properties of Interconnection Networks

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Abstract: In this paper we have taken various interconnection networks. In order to study properties of those networks we have derived their geometrical patterns from their respective incidence matrices. We have also applied some logical operations on incidence matrices to study various properties of interconnection networks.

Keyword: Interconnection Network, Topology, Sparse Matrix, Incidence Matrix.

I. INTRODUCTION

As we know network is a backbone of Internet today. Many topologies are present today for interconnection network and their study is vital for us. There are many features of network which we know and learn. In this paper we have tried to derive properties of Interconnection Networks with a new way using Matrix and logic gates operations. As we all know computer science is derived from Mathematics and Physics, so I have tried to link all three in a new way. First of all I will derive Incidence Matrix of various Interconnection Networks. This will give us a unique pattern in each matrix. These patterns may help us in distinguishing various properties of network. Also we will try to implement logical operation like AND operation on the row/column values of incidence matrix. At last we have given some formulae derived from geometrical patterns of network and we the help of these we will try to give a comparative study of properties of network.

II. INTERCONNECTION NETWORK

Multi-stage interconnection networks (MINs) are a class of high-speed computer networks usually composed of processing elements (PEs) on one end of the network and memory elements (MEs) on the other end, connected by switching elements (SEs). The switching elements themselves are usually connected to each other in stages, hence the name.

Such networks include networks, omega networks, delta networks and many other types. MINs are typically used in high- performance or parallel computing as a low-latency interconnection (as opposed to traditional packet switching networks), though they could be implemented on top of a packet switching network. Though the network is typically used for routing purposes, it could also be used as a co-processor to the actual processors for such uses as sorting; cyclic shifting, as in a perfect shuffle network; and bitonic sorting.[1]

III. TOPOLOGICAL PROPERTIES:

The following properties are associated with interconnection networks [7]:

a. *Topology:* It indicates how the nodes a network are organized. Various topologies are discussed in next section.
b. *Network Diameter*: It is the minimum distance between the farthest nodes in a network. The distance is measured in terms of number of distinct hops between any two nodes. In matrix,

   ND = total number of 1s diagonally/2
   Exception is in Pyramid,
   ND = total number of 1s diagonally
c. *Node degree*: Number of edges connected with a node is called node degree. If the edge carries data from the node, it is called out degree and if this carries data into the node it is called in degree.
   In matrix, number of 1s in a row or column represents node degree of that node.
d. *Bisection Bandwidth:* Number of edges required to be cut to divide a network into two halves is called bisection bandwidth.
   Matrix patterns easily represent bisection width of network,
e. *Data Routing Functions:* The data routing functions are the functions which when executed establishes the path between the source and the destination. In dynamic interconnection networks there can be various interconnection patterns that can be generated from a single network. This is done by executing various data routing functions. Thus data routing operations are used for routing the data between various processors. The data routing network can be static or dynamic static network.
f. *Static and Dynamic Interconnection Network:* In a static network the connection between input and output nodes is fixed and cannot be changed. Static interconnection network cannot be reconfigured. The examples of this type of network are linear array, ring,

chordal ring, tree, star, fat tree, mesh, tours, systolic arrays, and hypercube. This type of interconnection networks are more suitable for building computers where the communication pattern is more or less fixed, and can be implemented with static connections. In dynamic network the interconnection pattern between inputs and outputs can be changed. The interconnection pattern can be reconfigured according to the program demands. Here, instead of fixed connections, the switches or arbiters are used. Examples of such networks are buses, crossbar switches, and multistage networks. The dynamic networks are normally used in shared memory(SM) multiprocessors.

g. *Dimensionality of Interconnection Network:* Dimensionality indicates the arrangement of nodes or processing elements in an interconnection network. In single dimensional or linear network, nodes are connected in a linear fashion; in two dimensional network the processing elements (PE's) are arranged in a grid and in cube network they are arranged in a three dimensional network.

IV. SPARSE MATRIX

We have considered Sparse Matrix to study patterns of multi-core architectures. A sparse Matrix is a two-dimensional array having the value of majority elements as null. [9] Following is a sparse matrix where '*' denotes the elements having non-null values.

$$\begin{bmatrix} \_ & * & * & * & * \\ * & \_ & * & \_ & * \\ * & * & \_ & * & * \\ * & \_ & * & \_ & * \\ * & * & * & * & \_ \end{bmatrix}$$

Fig 1. A Sparse Matrix

V. MATRIX REPRESENTATION OF VARIOUS INTERCONNECTION NETWORKS

First of all we would derive an incidence matrix of given Interconnection Network by using following formula-

$$A_{ij} \begin{cases} 1 \; if \; node \; is \; connected \; to \; itself \; or \; to \; another \; node \\ 0 \; else \end{cases}$$

Where A represents a matrix with i number of rows and j number of columns.

The Incidence Matrix so derived is always a square sparse matrix since number of nodes for both column and rows are equal and most of the values are 0. Following are the Incidence Matrix so obtained of various Interconnection Networks. We have noticed unique patterns in each matrix. [11]

a) Mesh

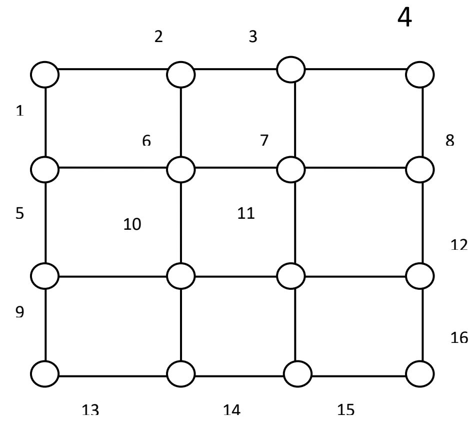


Figure 2. Mesh Topology

i\j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0
2	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0
3	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0
4	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0
5	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0
6	0	1	0	0	1	0	1	0	0	1	0	0	0	0	0	0
7	0	0	1	0	0	1	0	1	0	0	1	0	0	0	0	0
8	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0	0
9	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0
10	0	0	0	0	0	1	0	0	1	0	1	0	0	1	0	0
11	0	0	0	0	0	0	1	0	0	1	0	1	0	0	1	0
12	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	1
13	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0
14	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1	0
15	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1
16	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0

Figure 3. Pattern derived in Incidence Matrix of Mesh Topology (Fig.2).

b) Pyramid

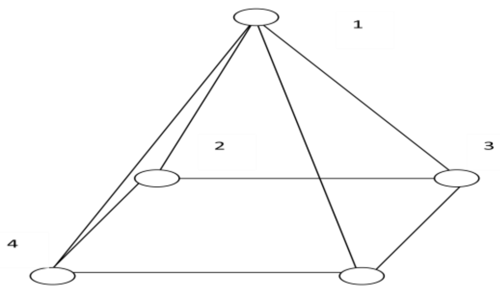


Figure 4. Pyramid Topology

i\j	1	2	3	4	5
1	0	1	1	1	1
2	1	0	1	1	0
3	1	1	0	0	1
4	1	1	0	0	1
5	1	0	1	1	0

Figure 5. Pattern derived in Incidence Matrix of Pyramid Topology (Fig.4).

c) *Torus*

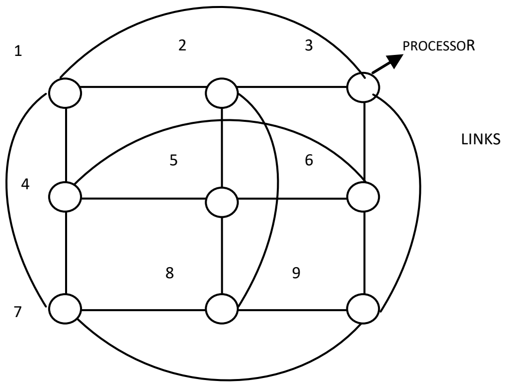


Figure. 6. Torus Topology

i\j	1	2	3	4	5	6	7	8	9
1	0	1	1	1	0	0	1	0	0
2	1	0	1	0	1	0	0	1	0
3	1	1	0	0	0	1	0	0	1
4	1	0	0	0	1	1	1	0	0
5	0	1	0	1	0	1	0	1	0
6	0	0	1	1	1	0	0	0	1
7	1	0	0	1	0	0	0	1	1
8	0	1	0	0	1	0	1	0	1
9	0	0	1	0	0	1	1	1	0

Figure.7. Pattern derived in Incidence Matrix of Torus Topology (Fig.6).

d) *Hypercube*

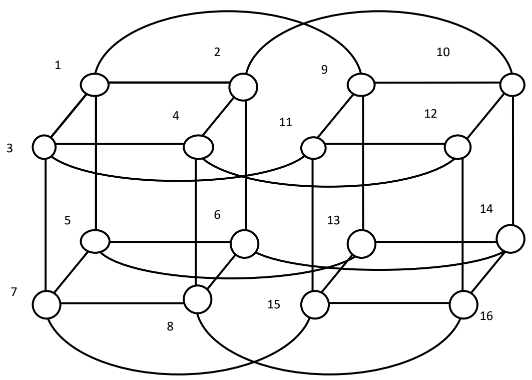


Figure. 8. Hypercube Topology

i\j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0
2	1	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0
3	1	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0
4	0	1	1	0	0	0	0	1	0	0	0	1	0	0	0	0
5	1	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0
6	0	1	0	0	1	0	0	1	0	0	0	0	0	1	0	0
7	0	0	1	0	1	0	0	1	0	0	0	0	0	0	1	0
8	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	1
9	1	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0
10	0	1	0	0	0	0	0	0	1	0	0	1	0	1	0	0
11	0	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0
12	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	1
13	0	0	0	0	1	0	0	0	1	0	0	0	0	1	1	0
14	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1
15	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	1
16	0	0	0	0	0	0	0	1	0	0	0	1	0	1	1	0

Figure.9. Pattern derived in Incidence Matrix of Hypercube Topology (Fig.8).

e) *Butterfly*

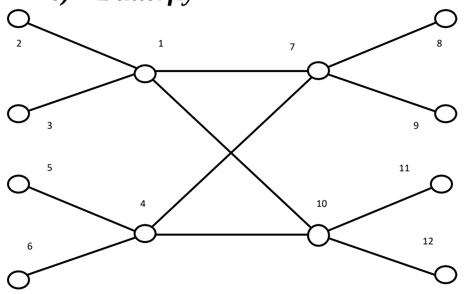


Figure. 10. Butterfly Topology

i\j	1	2	3	4	5	6	7	8	9	10	11	12
1	0	1	1	0	0	0	1	0	0	1	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	1	1	0	0	1	0	0
5	0	0	0	1	0	0	0	0	0	0	0	0
6	0	0	0	1	0	0	0	0	0	0	0	0
7	1	0	0	1	0	0	0	1	1	0	0	0
8	0	0	0	0	0	0	1	0	0	0	0	0
9	0	0	0	0	0	0	1	0	0	0	0	0
10	1	0	0	1	0	0	0	0	0	0	1	1
11	0	0	0	0	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	0	1	0	0

Fig.11. Pattern derived in Incidence Matrix of Butterfly Topology (Fig.10).

f) **Fat Tree**

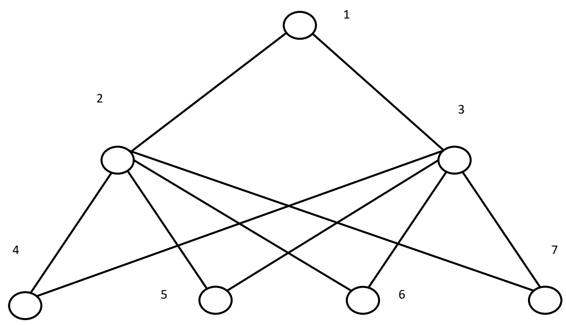


Figure. 12. Fat Tree Topology

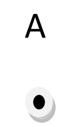
i\j	1	2	3	4	5	6	7
1	0	1	1	0	0	0	0
2	1	0	0	1	1	1	1
3	1	0	0	1	1	1	1
4	0	1	1	0	0	0	0
5	0	1	1	0	0	0	0
6	0	1	1	0	0	0	0
7	0	1	1	0	0	0	0

Figure. 13. Pattern derived in Incidence Matrix of Mesh (Fig.2).

VI. EVALUATION OF 'AND' AND 'OR' MATRIX

Here, we have taken network starting from node one/two and derived its incidence matrix. After that we have applied AND and OR logical operations on row-wise values of matrix.

1. Consider a Single Node (named A):



Incidence Matrix:

	A
A	1

2. Consider two nodes connected to each other in a line:

Incidence Matrix:

	A	B
A	0	1
B	1	0

A. *AND Matrix Evaluation: i is the number of row and j is the number of column:*

When i=j=1:

	0	1
0	0	0
1	0	1

When i=j=2

	1	0
1	1	0
0	0	0

When i=1 and j=2

	0	1
1	0	1
0	0	0

When i=2 and j=1

	1	0
0	0	0
1	1	0

B. *OR Matrix Evaluation: I is the number of row and j is the number of column:*

When i=j=1

	0	1
0	0	1
1	1	1

When i=j=2

	1	0
1	1	1
0	1	0

When i=1 and j=2

	0	1
1	1	1
0	0	1

When i=2 and j=1

	1	0
0	1	0
1	1	1

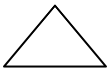
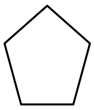
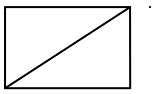
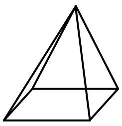
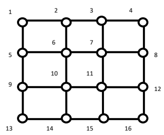
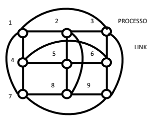
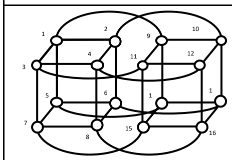
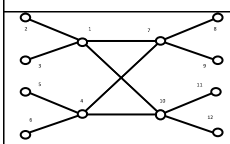
Similarly, we can implement AND and OR operations on matrices of other network topologies. We can also implement other logical operations like XOR and implication on incidence matrix of respective network.

VII. FORMULA DERIVED FROM INCIDENCE MATRIX

We have implemented AND and OR operation on other network topologies and on studying them we get the following formulae. We have arranged these formulae in tabular form in the following Table 1.We have also given evaluated values of various network topologies.

Table 1.Values obtained of various Interconnection Networks from derived Formula.

Graphical structure	Total number of edges(E)	Total number of vertices (V)	Total number of 1s in Incidence matrix (T_1) = E*2	Total Number of 1 in AND Matrix (T_{1A}) = E_C^2	Total Number of 1 in OR matrix (T_{1R}) = E+V+n

	1	2	1*2=2	1^2 = 1	1+2+0 = 3
	3	3	6	4	8
	4	4	8	4	12
	5	5	10	4	16
	5	4	10	4 or 9	13
	8	5	16	16 or 9	19
	24	16	24	4 or 9 or 16	68
	17	9	34	16	40
	32	16	64	16	76
	12	12	24	9 or 16	44

VIII. RELATED WORK

Study of topological properties of various interconnection networks is work of research in which we try to study properties of network to make communication more efficient between processors. Many research works has already been conducted related to this. We are trying to study them with new perspective so that we can get some new ways for communication or connection between processors. Following are some works already have been done in this field:

a. Study of Topological Property of Interconnection Networks and its Mapping to Sparse Matrix by Rakesh Kumar Katare, Narendra S. Chaudhari. [14]
b. Vector Operation on Nodes of Perfect Difference Network Using Logical Operators by Rakesh Katare.[13]
c. A Comparative Study of Hypercube and Perfect Difference Network for Parallel and Distributed System and its Application to Sparse Linear System by R.K. Katare and N.S. Chaudhari.[12]

IX. CONCLUSION AND FUTURE SCOPE

Here, we have given various unique patterns obtained from Incidence matrix of various network topologies. We have shown how to implement AND and OR logical operations on network topologies and derived AND value Matrix and OR value Matrix of those topologies. From geometrical patterns and logical operations we have derived some formulae which helped us in studying various properties of network topology. These formulae give us an idea of number of connections and nodes in a network. Also help us in getting degree of a node. On studying unique patterns , number of 1s and 0s in matrices we can give answers to various properties of network. We are trying to extend this extend this work in this direction. Also we can further take this work to next level by evaluating XOR, NAND, NOR or implication value Matrices and can study them with respect to our topologies. Even we can turn this towards Boolean algebra and can try to evaluate other Boolean expressions if possible.

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Ms. Pinki Sharma pursued M.Phil. in Computer Science from Awadhesh Pratap Singh University Rewa MP, India in 2013 and is currently pursuing Ph.D.She has published 2 research papers in reputed international journals and conferences and it is also available online. Her main research work focuses on study of Geometrical Patterns of Parallel and Distributed Network System. She has 12 years of teaching experience and 4 years of Research Experience.

Mr Rakesh Kumar Katare pursued Ph.D. in Computer Science and currently working as Professor in Department of Computer Science Awadhesh Pratap Singh University Rewa MP, India. He has published more than 15 research papers in reputed international journals and conferences including IEEE and it is also available online. His main research work focuses on Parallel and Distributed Algorithms, Network Topologies and Perfect Difference Network. He has more than 25 years of teaching experience and more than 10 years of Research Experience.

Dr. Reshma Begum pursued Ph.D. from Awadhesh Pratap Singh University Rewa MP, India in 2016. She is currently working as Assistant Professor in Government PG College, Seoni MP, India. She has published 5 research papers in reputed international journals and conferences and it is also available online. She has 12 years of teaching experience and 8 years of Research Experience.