# New Method to find initial basic feasible solution of Transportation Problem using MSDM 

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#### Abstract

The optimization processes in mathematics, computer science and economics are solved effectively by choosing the best element from set of available alternatives elements. The most important and successful applications in the optimization refer to transportation problem, that is a special class of the linear programming in the operation research. The main objective of transportation problem solution methods is to minimize the cost or the time of transportation. Most of the currently used methods for solving transportation problems are trying to reach the optimal solution, whereby, most of these methods are considered complex and very expansive in term of the execution time. Finding an initial basic feasible solution is the prime requirement to obtain an optimal solution for the transportation problems. In this paper, a new method named minimum supply \& demand method is proposed to find an initial basic feasible solution for the transportation problems. The method is also illustrated with numerical examples.


Keywords - Transportation Problem(TP), Transportation Cost(TC), Initial Basic Feasible Solution (IBFS), Optimal Solution, Vogel's approximation method(VAM), minimum supply \& demand method (MSDM).

## I. INTRODUCTION

Transportation problem is an important aspect in operations research domain for its wide applications in different real life problems and several areas of Science. The TP is a special type of Linear Programming Problem (LPP)[7], [11],[14],[15]. In 1941 Hitchcock developed the basic TP along with the constructive method of solution .Later in 1949 Koopmans discussed the problem in detail [14],[15] . Again in 1951 Dantzig formulated the TP as LPP and also provided the solution method and then by Charnes, Cooper and Henderson in 1953 .The Solution of TP is carried out in two phases ,first, an initial basic solution is found by various methods, and then the solution is tested for optimality and revised if necessary by using MOID or the Stepping Stone method[11],[14]. In this paper, the phase one has been focused in order to obtain a better initial basic feasible solution for the TP. Due to its importance, this problem has been studied by many researchers. Some of the methods for finding an initial basic feasible solution of TP are ASMmethod [5], Heuristic methods [1], Best candidates method [2], Proposed Approximation method and minimum transportation cost method [3], Total difference methods [13], ATM-method [6], Zero suffix method amd SAM method [8],maximum zero method [9] etc. The initial basic feasible solution (IBFS) is commonly obtained by using prominent methods namely "North West Corner", "Matrix Minima", "Least Cost Method", "Row Minima ", "Column

Minima" and "Vogel's Approximation
Method"[7],[11],[14],[15].
In this paper, a new method is proposed to find an initial basic feasible solution for the transportation problems.
Also the obtained IBFS is compared with the exiting prominent methods. The proposed method gives IBFS in shorter time.

## II. Mathematical Formulation of Transportation Problem

Let there be $m$ origins, $i$ th origin possessing $x_{i j}$ units of a certain product, whereas there be $n$ destinations with destination j requiring $\mathrm{b}_{\mathrm{j}}$ units. Let $\mathrm{c}_{\mathrm{ij}}$ be the cost of shipping one unit product from ith origin to j th destination. Let $\mathrm{x}_{\mathrm{ij}}$ be the number of units shipped from ith origin to jth destination. The problem is to determine non negative values of $\mathrm{x}_{\mathrm{ij}}$ satisfying the constraints:
$\sum_{j=1}^{n} \mathrm{x}_{\mathrm{ij}}=\mathrm{a}_{\mathrm{i}}, i=1,2,3, \ldots, m$ (availability constraints)
$\sum_{i=1}^{m} x_{i j}=b_{j}$ for $j=1,2,3, \ldots, n$ (requirment constraints)
And minimizing the total transportation cost =
$\sum_{i=1}^{m} \sum_{j=}^{n} x_{i j} \cdot c_{i j}$

## Tabular Representation of TP:

Suppose there are $\mathbf{m}$ facories and $\mathbf{n}$ warehouse.

| $\begin{aligned} & \mathbf{W} \rightarrow \\ & \mathbf{F} \downarrow \end{aligned}$ | W1 | W2 | ... | Wj | ... | Wn | F.c. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F1 | $\mathrm{C}_{11}$ | $\mathrm{C}_{12}$ | $\cdots$ | $\mathrm{C}_{1 \mathrm{j}}$ | ... | $\mathrm{C}_{1 \mathrm{n}}$ | $\mathrm{a}_{1}$ |
| F2 | $\mathrm{C}_{21}$ | $\mathrm{C}_{22}$ | $\cdots$ | $\mathrm{C}_{2 \mathrm{j}}$ | $\cdots$ | $\mathrm{C}_{2 \mathrm{n}}$ | $\mathbf{a}_{2}$ |
| : |  |  |  |  |  |  |  |
| Fi | $\mathrm{C}_{\text {i1 }}$ | $\mathrm{C}_{\mathrm{i} 2}$ | ... | $\mathrm{C}_{\mathrm{ij}}$ | ... | $\mathrm{C}_{\text {in }}$ | $\mathbf{a}_{\text {i }}$ |
| : |  |  |  |  |  |  |  |
| Fm | $\mathbf{C}_{\mathrm{m} 1}$ | $\mathrm{C}_{\mathrm{m} 2}$ | ... | $\mathrm{C}_{\mathrm{mj}}$ | ... | $\begin{aligned} & \mathbf{C}_{\mathrm{m}} \\ & \mathrm{n} \end{aligned}$ | $\mathbf{a}_{\mathrm{m}}$ |
| W.R. | $\mathrm{b}_{1}$ | $\mathrm{b}_{2}$ | ... | $\mathbf{b}_{\mathrm{j}}$ | ... | $\mathrm{b}_{\mathrm{n}}$ | $\sum_{i=1}^{m} a i=\sum_{j=1}^{n} b j$ |

(W:warehouse, F:factory, W.R. :warehouse requirement, F.C.:factory capacity )

Some Definitions :
Balanced TP : If $\quad \sum_{i=1}^{m} a_{i}=\sum_{j=1}^{n} b_{j}$
then the TP is called balanced otherwise TP is unbalanced.
Feasible Solution (FS): A set of non-negative individual allocations $\left(\mathrm{x}_{\mathrm{ij}} \geq 0\right)$ which simultaneously removes deficiencies is called a feasible solution.
Basic Feasible Solution (IBFS): A feasible solution to a m origins, $n$ destinations problem is said to be basic if the number of positive allocations are $\mathrm{m}+\mathrm{n}-1$.
Optimal solution : A feasible solution is said to be optimal if it minimizes the total transportation cost [7],[11],[14],[15].

## III. NEW METHOD :Minimum supply \& demand and method

Step 1:In balanced TP ,select the row or column with minimum supply or demand .If there is tie ,the select the row or column that contains cell with minimum transportation cost (TC).so in selected row or column ,select a cell say ( $\mathrm{i}, \mathrm{j})$ with minimum TC.
Step 2 : Allocate $\mathrm{x}_{\mathrm{ij}}=$ minimum $\left(\mathrm{a}_{\mathrm{i}}, \mathrm{b}_{\mathrm{j}}\right)$ in the $(\mathrm{i}, \mathrm{j})$ cell.
I ) If $x_{i j}=b_{j}$, cross out the $j$ th column of Transportation table (TT) and decrease $a_{i}$ by $b_{j}$. Go to Step 1 .
II ) ) If $\mathrm{x}_{\mathrm{ij}}=\mathrm{a}_{\mathrm{i}}$,cross out the i th row of Transportation tabl (TT) and decrease $b_{j}$ by $a_{i}$. Go to Step 1 .
III ) ) If $x_{i j}=a_{i}=b_{j}$, cross out either the ith row or $j$ th column or both of Transportation table TT) and decrease $\mathrm{a}_{\mathrm{i}}$ by $b_{j}$.Repeat steps 1 and 2 for reduced TP until all the requirements are satisfied.

Note : Whenever the minimum cost in not unique ,make an arbitrary choice among the minimum.

## Numerical Examples

EX.1) Consider the transportation problem presented in the following table.

| Factory <br> (origins) | Destinations (warehouse) |  |  |  | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | W1 | W2 | W3 | W4 |  |
| F1 | 19 | 30 | 50 | 10 | 7 |
| F2 | 70 | 30 | 40 | 60 | 9 |
| F3 | 40 | 8 | 70 | 20 | 18 |
| Demand | 5 | 8 | 7 | 14 | 34 |

Solution by Minimum supply \&demand Method :

| Factory <br> (origins) | Destinations (warehouse) |  |  |  | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | W1 | W2 | W3 | W4 |  |
| F1 | $19[5]$ | 30 | 50 | 10 | $7 \rightarrow 2$ |
| F2 | 70 | 30 | 40 | 60 | 9 |
| F3 | 40 | 8 | 70 | 20 | 18 |
| Demand | $5 \rightarrow 0$ | 8 | 7 | 14 | 34 |


| Factory <br> (origins) | Destinations (warehouse) |  |  |  | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | W1 | W2 | W3 | W4 |  |
| F1 | $19[5]$ | 30 | 50 | $10[2]$ | $7 \rightarrow 2 \rightarrow 0$ |
| F2 | 70 | 30 | 40 | 60 | 9 |
| F3 | 40 | 8 | 70 | 20 | 18 |
| Demand | $5 \rightarrow 0$ | 8 | 7 | $14 \rightarrow 12$ | 34 |


| Factory <br> (origins) | Destinations (warehouse) |  |  |  | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | W1 | W2 | W3 | W4 |  |
| F1 | $19[5]$ | 30 | 50 | $10[2]$ | $7 \rightarrow 2 \rightarrow 0$ |
| F2 | 70 | 30 | $40[7]$ | 60 | $9 \rightarrow 2$ |
| F3 | 40 | 8 | 70 | 20 | 18 |
| Demand | $5 \rightarrow 0$ | 8 | $7 \rightarrow 0$ | $14 \rightarrow 12$ | 34 |


| Factory <br> (origins) | Destinations (warehouse) |  |  |  | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | W1 | W2 | $\mathbf{W 3}$ | $\mathbf{W 4}$ |  |
| F1 | $19[5]$ | 30 | 50 | $10[2]$ | $7 \rightarrow 2 \rightarrow 0$ |
| F2 | 70 | $30[2]$ | $40[7]$ | 60 | $9 \rightarrow 2 \rightarrow 0$ |
|  |  |  |  |  |  |
| F3 | 40 | 8 | 70 | 20 | 18 |
| Demand | $5 \rightarrow 0$ | $8 \rightarrow 6$ | $7 \rightarrow 0$ | $14 \rightarrow 12$ | 34 |


| Factory <br> (origins) | Destinations (warehouse) |  |  |  | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | W1 | $\mathbf{W 2}$ | $\mathbf{W 3}$ | $\mathbf{W 4}$ |  |
| F1 | $19[5]$ | 30 | 50 | $10[2]$ | $7 \rightarrow 2 \rightarrow 0$ |
| F2 | 70 | $30[2]$ | $40[7]$ | 60 | $9 \rightarrow 2 \rightarrow 0$ |
| F3 | 40 | $8[6]$ | 70 | 20 | $18 \rightarrow 12$ |
| Demand | $5 \rightarrow 0$ | $8 \rightarrow 6 \rightarrow$ <br> 0 | $7 \rightarrow 0$ | $14 \rightarrow 12$ | 34 |


| Factory <br> (origins) | Destinations (warehouse) |  |  |  | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | W1 | W2 | W3 | W4 |  |
| F1 | $19[5]$ | 30 | 50 | $10[2]$ | $7 \rightarrow 2 \rightarrow 0$ |
| F2 | 70 | $30[2]$ | $40[7]$ | 60 | $9 \rightarrow 2 \rightarrow 0$ |
|  |  |  |  |  |  |
| F3 | 40 | $8[6]$ | 70 | $20[12]$ | $18 \rightarrow 12 \rightarrow 0$ |
| Demand | $5 \rightarrow 0$ | $8 \rightarrow 6 \rightarrow 0$ | $7 \rightarrow 0$ | $14 \rightarrow 12 \rightarrow 0$ | 34 |

The initial basic feasible solution by minimum supply \& demand method in single TP:

| Factory <br> (origins) | Destinations (warehouse) |  |  |  | Supply |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | W1 | W2 | W3 | W4 |  |
| F1 | $19[5]$ | 30 | 50 | $10[2]$ | $7-5=2 \leftarrow$ |
| F2 | 70 | $30[2]$ | $40[7]$ | 60 | $9-7=2 \leftarrow$ |
| F3 | 40 | $8[6]$ | 70 | $20[12]$ | $18-6=12-$ <br> $12=0$ |
| Demand | 5 <br> $\uparrow$ | $8-2=6$ <br> $\uparrow$ | 7 <br> $\uparrow$ | $14-2=12$ <br> $\uparrow$ | 34 |

So,Minimum TC $=5 \times 19+2 \times 10+2 \times 30+7 \times 40+6 \times 8+12 \times 20$ $=743$.
EX.2) Consider the transportation problem .

| (origins) | Supply |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | Destinations |  |  |  |
|  | D1 | D2 | D3 |  |
| O1 | 50 | 30 | 220 | 1 |
| O2 | 90 | 45 | 170 | 3 |
| O3 | 250 | 200 | 50 | 4 |
| Demand | 4 | 2 | 2 |  |

Transportation costs per unit of production from particular three origins to different three destinations are given in the above Table.

## Solution by Minimum Supply \&Demand Method :

| Origins | Destinations |  |  | Supply |
| :--- | :--- | :--- | :--- | :--- |
|  | D1 | $\mathbf{D 2}$ | $\mathbf{D 3}$ |  |
| O1 | $\mathbf{5 0}$ | $\mathbf{3 0}[\mathbf{1}]$ | $\mathbf{2 2 0}$ | $\mathbf{1} \leftarrow$ |
| O2 | $\mathbf{9 0}[2]$ | $\mathbf{4 5}[\mathbf{1}]$ | $\mathbf{1 7 0}$ | $\mathbf{3 - 1 = 2} \leftarrow$ |
| O3 | $\mathbf{2 5 0 [ 2 ]}$ | $\mathbf{2 0 0}$ | $\mathbf{5 0}[2]$ | $\mathbf{4 - 2}=\mathbf{2} \leftarrow$ |
| Demand | $\mathbf{4 - 2}=\mathbf{2 - 2}=\mathbf{0}$ | $\mathbf{2 - 1 = 1}$ | $\mathbf{2}$ |  |
|  |  | $\uparrow$ | $\uparrow$ |  |

So ,Minimum TC $=1 \times 30+2 \times 90+1 \times 45+2 \times 250+2 \times 50=855$.

## Comparison Table :

| Ex. | Minimum TC by |  |  |  | TC of Optimal Solution by MODI method |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | VAM | NWCR | LCM | MSDM |  |
| 1 | 743 | 1015 | 814 | 743 | 743 |
| 2 | 820 | 820 | 855 | 855 | 820 |

## IV. RESULTS AND DISCUSSION

The comparison table shows a comparison of the TC obtained by MSDM and the existing prominent methods along with the optimal solution by means of the above two sample examples.The comparative study shows that the proposed method gives better or same result in comparison with the other existing methods. Th MSDM is simple and reliable .It is easy to compute, understand and thus saves time.

## V. CONCLUSION AND FUTURE SCOPE

In this study, I have proposed a new method, MSDM, for finding an initial basic feasible solution of transportation problems. The MSDM is easy and simple comparatively than VAM. The IBFS obtained by MSDM is either the optimal solution or closer to the optimal solution with minimum computation time. The lower computational complexity is because of less calculations in the proposed method. Therefore this method can be applied in real transportation problems.

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