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Cost-Profit Analysis of an Infinite Capacity Multi-server Markovian Feedback Queuing System with Reverse Balking

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Abstract - Balking is a customer behavior in which a customer upon arrival refuses to join the system if large number of customers are already present in the system. But in many businesses such as restaurants, healthcare, investment etc., it can be seen that the reverse of this phenomenon prevails. A large customer base acts as a motivating factor for newly arriving customers in such businesses with notion of getting better quality of service, affordability or both. This phenomenon is termed as Reverse Balking and it results in higher probability of a customer joining the system with respect to increasing customer base. This increasing probability of joining puts service facility under pressure. That in turn results in dissatisfactory and incomplete service at times. A dissatisfied customer may join the queue again for satisfactory service and is termed as a *feedback customer* in queuing literature. In order to frame an effective operational policy for such a system, it is essential to measure the performance of the system in advance. In this paper we combine above mentioned contemporary challenges of reverse balking and feedback to formulate a new multi-server infinite capacity feedback Markovian queuing system with reverse balking. The system is studied in steady-state. The necessary probability measures and measures of performance are derived. The sensitivity analysis of the model is presented. Later the cost model is developed and cost-profit analysis of the model is also presented. Algorithms are written in MATLAB and MS Excel for sensitivity analysis.

Key words: reverse balking, multi-server, queuing theory, feedback queue, infinite capacity.

I. INTRODUCTION AND LITERATURE SURVEY

In today's highly competitive global era, understanding customer behavior and designing strategies in advance to stay ahead of the competitors is of utmost priority for organizations. It is generally believed that more number of customers present in the system waiting for service acts as a discouraging factor for newly arriving customers and thus the probability of arriving customers joining such system becomes lesser with respect to increasing customer base. This phenomenon is studied by [1], [2] and [3] and is termed as balking in queuing literature. [4] studied multi-server queuing model with discouragement and [5] also studied single server queuing system with state dependent parameters in which he studied discouragement caused by increasing queue length for newly arriving customers. On the contrary [6] observed that in many businesses such as restaurants, healthcare, life insurance, investment etc. customers perceive that large customer base ensures better quality, value for money or both. As a result large customer base acts as a motivating factor for newly arriving customers and thus results in higher probability of a customer joining the system with large customer base.

Large customer base at times put service facility under pressure, which may result in dissatisfaction amongst certain customers and such customers may leave the system dissatisfied and rejoin the queue again for completion of service satisfactorily. These customers are termed as feedback customers in queuing literature. [7] studied feedback queue to determine the queue size and studied the distribution function of the average time a customer spent in the system. Feedback queues are also studied by [8] and [9] considered a feedback queue and incorporated the phenomenon of customer impatience. Infinite capacity reverse balking queuing system is presented in [10] and later extended with the concept of feedback in [11].

In today's highly competitive global era, understanding customer behavior and designing strategies in advance to stay ahead of the competitors is of utmost priority for organizations. It is evident from the literature review that no researcher has studied infinite capacity feedback queuing model with reverse balking. Owing to the valid aspect of reverse balking with feedback customers we develop an infinite capacity feedback queuing model with reverse balking and further developed the cost model and analyzed the developed cost-profit queuing model in this paper. Rest of the paper is organized as follows. Model assumptions and formulation of model are presented in section 2 and section 3 deals with steady-state solution of the model. Measures of performance are derived in section 4 and sensitivity analysis of the model with respect to performance measures is presented in section 5. Section 6 deals with cost-profit

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analysis of the model by developing a cost model and sensitivity analysis is also performed. Finally, conclusion and future scope are given in section 7.

II. MATHEMATICAL MODEL FORMULATION

Model is formulated under the following assumptions:

(i) The arrivals occur in accordance to Poisson process with parameter λ.

(ii) A dissatisfied customer may rejoin the queue for service with probability q and may leave the queue satisfactorily with probability $p = 1 - q$.

(iii) Service times are exponentially distributed with parameter μ.

(iv) Queue discipline is first come first served.

(v) There are c servers through which the service is provided.

(vi) Capacity of the system is infinite.

(vii) Probability of reverse balking at $n = 0$ is $q' = (1 - p')$. And for $n \geq 0$ an arriving customer may reverse balk (refuse to join the system) with the probability $\left(\frac{1}{n+1}\right)$ may not reverse balk (join the system) with the probability $\left(\frac{n}{n+1}\right)$.

Differential difference equations governing the model are given by:

$$\frac{d}{dt}P_0(t) = -\lambda p' P_0(t) + \mu p P_1(t) \;; n = 0 \quad (1)$$

$$\frac{d}{dt}P_1(t) = \lambda p' P_0(t) - \left(\frac{\lambda}{2} + \mu p\right) P_1(t) + 2\mu p P_2(t) \quad ; n = 1 \quad (2)$$

$$\frac{d}{dt}P_n(t) = \lambda\left(\frac{n-1}{n}\right) P_{n-1}(t) - \left(\frac{\lambda n}{n+1} + n\mu p\right) P_n(t) + (n+1)\mu p P_{n+1}(t); 1 \leq n \leq c-1 \quad (3)$$

$$\frac{d}{dt}P_n(t) = \lambda\left(\frac{n-1}{n}\right) P_{n-1}(t) - \left(\frac{\lambda n}{n+1} + c\mu p\right) P_n(t) + c\mu p P_{n+1}(t) \quad ; n \geq c \quad (4)$$

In steady state, $as\ t \to \infty, P_n(t) = P_n$ and therefore, $\frac{d}{dt}P_n(t) = 0\ as\ t \to \infty$ and hence, equations (1) – (4) become:

$$0 = -\lambda p' P_0 + \mu p P_1 \quad ; n = 0 \quad (5)$$

$$0 = \lambda p' P_0 - \left(\frac{\lambda}{2} + \mu p\right) P_1 + 2\mu p P_2 \quad ; n = 1 \quad (6)$$

$$0 = \lambda\left(\frac{n-1}{n}\right) P_{n-1} - \left(\frac{\lambda n}{n+1} + n\mu p\right) P_n + (n+1)\mu p P_{n+1} \;; 2 \leq n \leq c-1 \quad (7)$$

$$0 = \lambda\left(\frac{n-1}{n}\right) P_{n-1} - \left(\frac{\lambda n}{n+1} + c\mu p\right) P_n + c\mu p P_{n+1} \quad ; n \geq c \quad (8)$$

III. STEADY-STATE SOLUTION

On solving (5) - (8) iteratively we get;

$$P_n = \Pr\{n \text{ customers in the system}\}$$

$$= \begin{cases} \dfrac{1}{n(n!)}\left(\dfrac{\lambda}{\mu p}\right)^n p' P_0 & 1 \leq n \leq c-1 \\ \dfrac{c^c}{n(c!)}\left(\dfrac{\lambda}{c\mu p}\right)^n p' P_0; & n \geq c \end{cases} \quad (9)$$

Using condition of normality $\sum_{n=0}^{\infty} P_n = 1$, we get

$$P_0 = Pr\{\text{system is empty}\}$$

$$= \left[1 + p' \sum_{n=1}^{c-1} \frac{1}{n(n!)}\left(\frac{\lambda}{\mu p}\right)^n + \frac{c^c}{c!} p' \left\{-\log\left(1 - \frac{\lambda}{c\mu p}\right) - \sum_{n=1}^{c-1} \frac{1}{n}\left(\frac{\lambda}{c\mu p}\right)^n\right\}\right]^{-1} \quad (10)$$

IV. MEASURES OF PERFORMANCE

1. Probability that a customer upon arrival finds all servers busy (P_b):

$$P_b = \Pr\{\text{all servers are busy}\} = \sum_{n=c}^{\infty} P_n$$

$$= \frac{c^c}{c!}\left\{-\log\left(1 - \frac{\lambda}{c\mu p}\right) - \sum_{n=1}^{c-1} \frac{1}{n}\left(\frac{\lambda}{c\mu p}\right)^n\right\} p' P_0 \quad (11)$$

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2. Length of the queue (L_q):

$$L_q = \sum_{n=c}^{\infty}(n-c)P_n$$

$$= \frac{c^c}{c!}\left(\frac{\lambda}{c\mu p}\right)^c \frac{1}{1-\frac{\lambda}{c\mu p}}\, p'P_0 - \frac{c^{c+1}}{c!}\left\{-\log\left(1-\frac{\lambda}{c\mu p}\right) - \sum_{n=1}^{c-1}\frac{1}{n}\left(\frac{\lambda}{c\mu p}\right)^n\right\} p'P_0 \qquad (12)$$

3. Average waiting time in the queue (W_q):

$$W_q = \frac{L_q}{\lambda} = \frac{1}{\lambda}\left[\frac{c^c}{c!}\left(\frac{\lambda}{c\mu p}\right)^c \frac{1}{1-\frac{\lambda}{c\mu p}}\, p'P_0 - \frac{c^{c+1}}{c!}\left\{-\log\left(1-\frac{\lambda}{c\mu p}\right) - \sum_{n=1}^{c-1}\frac{1}{n}\left(\frac{\lambda}{c\mu p}\right)^n\right\} p'P_0\right] \qquad (13)$$

4. Average waiting time in the system (W_s):

$$W_s = W_q + \frac{1}{\mu}$$

$$= \frac{1}{\lambda}\left[\frac{c^c}{c!}\left(\frac{\lambda}{c\mu p}\right)^c \frac{1}{1-\frac{\lambda}{c\mu p}}\, p'P_0 - \frac{c^{c+1}}{c!}\left\{-\log\left(1-\frac{\lambda}{c\mu p}\right) - \sum_{n=1}^{c-1}\frac{1}{n}\left(\frac{\lambda}{c\mu p}\right)^n\right\} p'P_0\right] + \frac{1}{\mu} \qquad (14)$$

5. Length of the system (L_s):

$$L_s = \lambda W_s$$

$$= \left[\frac{c^c}{c!}\left(\frac{\lambda}{c\mu p}\right)^c \frac{1}{1-\frac{\lambda}{c\mu p}}\, p'P_0 - \frac{c^{c+1}}{c!}\left\{-\log\left(1-\frac{\lambda}{c\mu p}\right) - \sum_{n=1}^{c-1}\frac{1}{n}\left(\frac{\lambda}{c\mu p}\right)^n\right\} p'P_0\right] + \frac{\lambda}{\mu} \qquad (15)$$

6. Average rate of reverse balking (R_b):

$$R_b = q'\lambda P_0 + \sum_{n=1}^{\infty}\left(\frac{\lambda}{n+1}\right)P_n$$

$$= q'\lambda P_0 + \sum_{n=1}^{c-1}\frac{1}{n(n+1)!}\left(\frac{\lambda}{\mu p}\right)^n p'\lambda P_0 + \frac{c^c}{c!}p'\lambda P_0\left[1+\frac{(1-x)\log(1-x)}{x} - \sum_{n=1}^{c-1}\frac{1}{n(n+1)}\left(\frac{\lambda}{c\mu p}\right)^n\right] \qquad (16)$$

where $x = \frac{\lambda}{c\mu p}$.

V. SENSITIVITY ANALYSIS

This section illustrates the variations in various performance measures with respect to variations in a particular parameter.

It can be observed from the plot below that with increase in average rate of arrival the length of the system as well as the length of the queue increases which puts service facility under pressure and as a result average rate of reverse balking also increases.

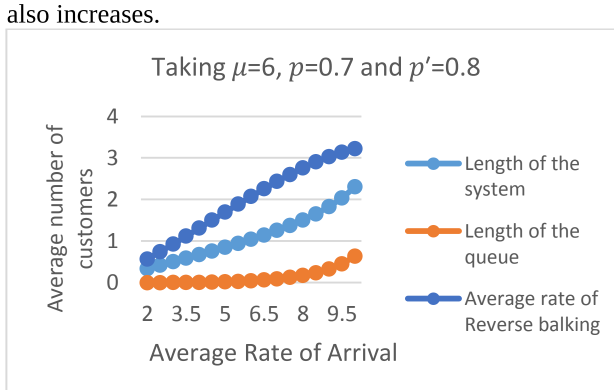

Figure 1 Variations in performance measures, L_s, L_q and R_b w.r.t. arrival rate (λ)

Similarly, it can be observed from the plot below that with increase in average rate of service the length of the system as well as the length of the queue decreases. Though average rate of reverse balking increases but there is a very slight variation in it.

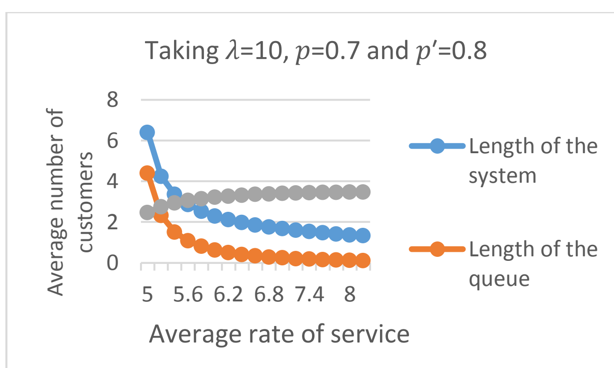

Figure 2 Variations in various performance measures, L_s, L_q and R_b w.r.t. service rate (μ)

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VI. COST-PROFIT ANALYSIS

This section deals with the economic analysis of the model by developing the functions of total expected cost (TEC), total expected revenue (TER) and total expected profit (TEP):

$$TEC = cC_s\mu + C_hL_s + C_bR_b + C_fqL_s$$

$$TER = R \times c \times \mu(1 - P_0)$$

$$TEP = TER - TEC$$

Where, C_s= Cost per service per unit time, C_h= holding cost per unit per unit time, C_b= Cost associated to each reverse balked unit per unit time, C_f= Cost associated to each feedback customer per unit time, R= Revenue earned per unit per unit time

Translating the above cost model in MS EXCEL and performing the sensitivity analysis for varying rates of arrival and service we obtain the following:

It is evident from the table below that the total expected profit increases with increase in average rate of arrival.

Table 1 Variation in TEC, TER and TEP with respect to λ

Average rate of arrival (λ)	Total Expected Cost (TEC)	Total Expected Revenue (TER)	Total Expected Profit (TEP)
2	369.99	542.80	172.81
2.5	372.91	644.74	271.82
3	375.93	736.95	361.03
3.5	379.01	820.77	441.77
4	382.14	897.30	515.16
4.5	385.30	967.45	582.15
5	388.48	1032.01	643.53
5.5	391.66	1091.66	700.00
6	394.84	1146.98	752.14
6.5	398.01	1198.48	800.47
7	401.17	1246.61	845.45
7.5	404.30	1291.78	887.47
8	407.44	1334.35	926.92
8.5	410.58	1374.69	964.11
9	413.76	1413.14	999.38
9.5	417.05	1450.04	1033.00
10	420.56	1485.81	1065.25

Source: simulated data, taking $\mu = 6, C_S = 20, C_f = 15, C_h = 5, C_b = 12, R = 100$

The table below shows that the total expected profit increases with increase in average rate of service.

Table 2 Variation in TEC, TER and TEP with respect to μ

Average rate of service (μ)	Total Expected Cost (TEC)	Total Expected Revenue (TER)	Total Expected Profit (TEP)
5	390.55	1361.33	970.79
5.2	385.52	1382.88	997.36
5.4	391.21	1407.69	1016.47
5.6	399.95	1433.57	1033.62
5.8	409.95	1459.74	1049.79
6	420.56	1485.81	1065.25
6.2	431.53	1511.61	1080.08
6.4	442.70	1537.00	1094.30
6.6	454.01	1561.95	1107.93
6.8	465.41	1586.40	1120.99
7	476.87	1610.34	1133.47
7.2	488.38	1633.77	1145.38
7.4	499.93	1656.68	1156.75
7.6	511.50	1679.08	1167.58
7.8	523.10	1700.97	1177.87
8	534.71	1722.37	1187.66
8.2	546.34	1743.28	1196.94

Source: simulated data taking $\lambda = 10, C_S = 20, C_f = 15, C_h = 5, C_b = 12, R = 100$

The table below shows that the total expected profit increases with increase in the reverse balking probability of joining the system.

Table 3 Variation in TEC, TER and TEP with respect to p'.

Reverse Balking Probability of joining the system when system is empty (p')	Total Expected Cost (TEC)	Total Expected Revenue (TER)	Total Expected Profit (TEP)
0.2	446.4328	975.1714	528.7386
0.25	441.4485	1073.5607	632.1121
0.3	437.5267	1150.9789	713.4522
0.35	434.3602	1213.4850	779.1248
0.4	431.7501	1265.0091	833.2590
0.45	429.5615	1308.2116	878.6501
0.5	427.7000	1344.9580	917.2580
0.55	426.0973	1376.5948	950.4975
0.6	424.7030	1404.1185	979.4154
0.65	423.4790	1428.2822	1004.8032
0.7	422.3957	1449.6658	1027.2701

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0.75	421.4303	1468.7229	1047.2927
0.8	420.5645	1485.8138	1065.2493
0.85	419.7837	1501.2277	1081.4440
0.9	419.0758	1515.1999	1096.1240
0.95	418.4313	1527.9237	1109.4924
1	417.8418	1539.5591	1121.7173

Source: simulated data taking $\lambda = 10, C_S = 20, C_f = 15, C_h = 5, C_b = 12, R = 100$

VII. CONCLUSIONS AND FUTURE SCOPE

In this paper we developed a multi-server infinite feedback queuing system with reverse balking. Model is solved in steady-state iteratively. The necessary measures of performance of the model are obtained. Cost model of newly developed model is derived and cost-profit analysis is performed. Numerical illustration of the model is presented after writing an algorithm in MATLAB and MS Excel. The results are of immense use for any firm operating under mentioned challenges.

The future scope of the model lies in the idea of studying it with another parameters of customer behaviors. Non-Markovian models of reverse balking may also be studied. The model can further be optimized for service time and arrival rate.

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