

## On Neighbourhood Difference Cordial Labeling of Networks

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DOI: <https://doi.org/10.26438/ijcse/v7i3.538543> | Available online at: [www.ijcseonline.org](http://www.ijcseonline.org)

Accepted: 13/Mar/2019, Published: 31/Mar/2019

**Abstract**— Graph labeling is the mapping of graph elements like vertices or edges or both to set of integers under some conditions. In a binary labeling to vertices, if we assign the edge label as the difference of the label of end vertices and if the difference of the number of vertices with label 0 and 1 and difference of edges with label 0 and 1 is at most 1, then the labeling is cordial labeling. If the mapping is a one to one mapping from the vertices to the labels from  $\{1, 2, \dots, p\}$ , where  $p$  is the cardinality of the vertex set, edge label as in the cordial labeling and if the difference of the number of edges labeled with 1 and not labeled with 1 is at most 1, then the labeling is difference cordial labeling. Neighborhood difference cordial labeling is a variation of difference cordial labeling, if the difference of the number of edges labeled with 1 and not labeled with 1 at each vertex is at most 1 then the labeling is neighbourhood difference cordial labeling. In this paper we investigate the neighbourhood difference cordial labeling of honey comb network, butterfly network, benes network and grid network.

**Keywords**— Honey comb network, butterfly network, benes network, grid and difference cordial label.

### I. INTRODUCTION

Cordial labeling was established by Cahit [1] in 1987 and Ponraj et al [2] introduced difference cordial labeling of a  $G(p, q)$  graph in 2013 and discussed about the difference cordial labeling of path, cycle, complete graph, star, complete bipartite graph, bistar, wheel, fan, gear graph, web graph and helm graph. In 2015 Seoud and Salman [3] studied difference cordial labeling of various types of ladders and one point union of graphs. Xavier et al [5] initiated the study of neighbourhood difference cordial labeling of graphs. In this paper we are going to discuss about neighbourhood difference cordiality of few networks.

### II. PRELIMINARIES

**Definition 1:** A honey comb network  $HC(n)$  of size  $n$  is obtained from  $HC(n - 1)$  by adding a layer of hexagon around the boundary of  $HC(n - 1)$ , where  $HC(1)$  is a hexagon.

**Definition 2:**[4] The  $n$  dimensional butterfly network, denoted by  $BF(n)$ , has a vertex set  $V = \{(x; i); x \in V(Q_n), 0 \leq i \leq n\}$ . Two vertices  $(x; i)$  and  $(y; j)$  are linked by an edge in  $BF(n)$  if and only if  $j = i + 1$  and either

- (i).  $x = y$  or
- (ii).  $x$  differs from  $y$  in precisely the  $j^{\text{th}}$  bit.

For  $x = y$ , the edge is said to be a straight edge. Otherwise the edge is a cross edge. For fixed  $i$  the vertex  $(x; i)$  is a vertex on level  $i$ .

**Definition 3:** The  $n$  dimensional benes network  $BB(n)$  consists of back to back butterfly network and it has  $2n + 1$  levels,  $(2n + 1)2^n$  vertices and  $n2^{n+2}$  edges.

**Definition 4:** Cartesian product of  $G \times H$  of graphs  $G$  and  $H$  is a graph with vertex set  $V(G) \times V(H)$  and two vertices  $(u, v)$  and  $(u_1, v_1)$  in  $G \times H$  are adjacent if  $u = u_1$  and  $v$  is adjacent to  $v_1$  in  $H$  or  $v = v_1$  and  $u$  is adjacent to  $u_1$  in  $G$ .

**Definition 5:** A path is an alternating sequence of distinct vertices and edges.

**Definition 6:** Let  $P_n$  denote the path on  $n$  vertices. For  $m, n \geq 2$ ,  $P_m \times P_n$  is defined as the two dimensional grid with  $m$  rows and  $n$  columns and is denoted by  $M_{m \times n}$ . Any vertex in  $i^{\text{th}}$  row and  $j^{\text{th}}$  column is denoted by  $V_{ij}$ .

**Definition7:** Difference cordial labeling: Let  $G(p, q)$  be a graph. Let  $f: V(G) \rightarrow \{1, 2, 3 \dots p\}$  be a function. For each  $uv$  assign the label  $|f(u) - f(v)|$ .  $f$  is called difference cordial labeling if  $f$  is one to one and the absolute difference of number of edges labeled with 1 and edges not labeled with 1 is at most 1. Any graph with difference cordial labeling is called difference cordial graph.

Honey comb coordinate system: The coordinate system honey comb network is as given below

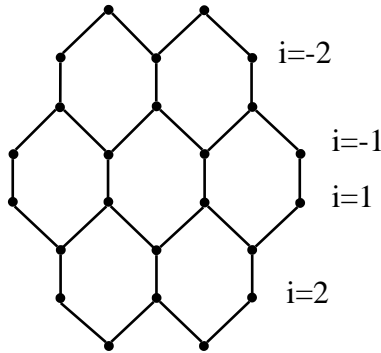


Fig 1: Coordinate system of HC (2)

**Neighbourhood difference cordial labelling:**

Let  $G = (V, E)$  be a graph where  $V$  and  $E$  are the vertex and edge sets of  $G$ . Let  $f: V \rightarrow \{1, 2, 3, \dots, p\}$  be a function. For each edge  $uv$  assign the label  $|f(u) - f(v)|$ .  $f$  is called a neighbourhood difference cordial labeling if  $f$  is one to one map and for every vertex  $v, |ef_v(1)|$  where

$ef_v(1)$  and  $ef_v(0)$  denote the number of edges incident with  $v$ , labeled with 1 and not labeled with 1 respectively. A graph with neighbourhood difference cordial labeling is called neighbourhood difference cordial graph.

**Theorem 1:** Anyhoney comb network  $HC(n)$  is neighborhood difference cordial graph.

**Proof:** A honey comb network of size  $n$   $HC(n)$  has  $6n^2$  vertices and  $9n^2 - 3n$  edges. In that  $6n$  vertices of degree 2 and  $6n^2 - 6n$  vertices are of degree 3.

Let  $V = \{v_1, v_2, v_3, \dots, v_p\}$

Define  $f: V \rightarrow \{1, 2, 3, \dots, 6n^2\}$

**Case 1: When  $k$  is positive**

$$f(v_{kj}) = 3n^2 - \sum_{i=1}^k (4n - (2i - 1) + j), \quad 1 \leq k \leq n - 2, 1 \leq j \leq (4n - (2k - 1))$$

At  $v_{kj}, 2 \leq k \leq n - 2, 1 \leq j \leq (4n - (2k - 2))$

$$\begin{aligned} &|f(v_{kj}) - f(v_{k(j+1)})| = \\ &\left| \left( 3n^2 - \sum_{i=1}^k (4n - (2i - 1) + j) \right) - \left( 3n^2 - \sum_{i=1}^k (4n - (2i - 1) + j + 1) \right) \right| = 1 \end{aligned}$$

**When  $j$  is even**

$$\begin{aligned} &|f(v_{kj}) - f(v_{(k+1)(j-1)})| \\ &= \left| \left( 3n^2 - \sum_{i=1}^k (4n - (2i - 1) + j) \right) - \left( 3n^2 - \sum_{i=1}^{k+1} (4n - (2i - 1) + j - 1) \right) \right| = 4n - 2(k + 1) \end{aligned}$$

**When  $j$  is odd**

$$\begin{aligned} &|f(v_{kj}) - f(v_{(k-1)(j+1)})| \\ &= \left| \left( 3n^2 - \sum_{i=1}^k (4n - (2i - 1) + j) \right) - \left( 3n^2 - \sum_{i=1}^{k-1} (4n - (2i - 1) + j + 1) \right) \right| = (4n - 2k) \end{aligned}$$

$$|ef_{v_{kj}}(1) - ef_{v_{kj}(0)}| = 1$$

At  $v_{1j}, 2 \leq j \leq 4n - 2$

$$|f(v_{1j}) - f(v_{1(j+1)})| = |(3n^2 - (4n - 1) + j) - (3n^2 - (4n - 1) + j + 1)| = 1$$

When  $j$  is even

$$\begin{aligned} &|f(v_{1j}) - f(v_{2(j-1)})| = |(3n^2 - (4n - 1) + j) - (3n^2 - (4n - 1) - (4n - 3) + j - 1)| \\ &= (4n - 4) \end{aligned}$$

When  $j$  is odd

$$|f(v_{1j}) - f(v_{(1-1)j})| = |(3n^2 - (4n - 1) + j) - (3n^2 + j)| = (4n - 1)$$

$$|ef_{v_{1j}}(1) - ef_{v_{1j}(0)}| = 1$$

**Case 2: When  $k$  is negative**

$$f(v_{(-1)j}) = 3n^2 + j, 1 \leq j \leq (4n - 1)$$

$$f(v_{kj}) = 3n^2 + \sum_{i=1}^{k-1} (4n - (2i - 1) + j), 2 \leq k \leq n - 2, 1 \leq j \leq (4n - (2k - 1))$$

At  $v_{kj}, 2 \leq k \leq n - 2, 1 \leq j \leq (4n - (2k - 1))$

**When  $j$  is even**

$$\begin{aligned} &|f(v_{kj}) - f(v_{k(j+1)})| \\ &= \left| \left( 3n^2 + \sum_{i=1}^k (4n - (2i - 1) + j) \right) - \left( 3n^2 + \sum_{i=1}^k (4n - (2i - 1) + j + 1) \right) \right| = 1 \end{aligned}$$

**When  $j$  is odd**

$$|f(v_{kj}) - f(v_{(k-1)(j+1)})|$$

$$= \left| \left( 3n^2 + \sum_{i=1}^{k-1} (4n - (2i-1) + j) \right) - \left( 3n^2 + \sum_{i=1}^{k-2} (4n - (2i-1) + j + 1) \right) \right| = (4n - 2(k-1))$$

$$|ef_{v_{kj}}(1) - ef_{v_{kj}}(0)| = 1$$

At  $v_{(-1)j}$ ,  $2 \leq j \leq 4n - 2$

$$|f(v_{(-1)j}) - f(v_{(-1)(j+1)})| = |(3n^2 + j) - (3n^2 + j + 1)| = 1$$

**When  $j$  is even**

$$|f(v_{(-1)j}) - f(v_{(-2)(j-1)})| = |(3n^2 + j) - (3n^2 + (4n - 1) + j - 1)| = (4n - 2)$$

**When  $j$  is odd**

$$|f(v_{1j}) - f(v_{(-1)j})| = |(3n^2 - (4n - 1) + j) - (3n^2 + j)| = (4n - 1)$$

$$|ef_{v_{(-1)j}}(1) - ef_{v_{(-1)j}}(0)| = 1$$

**Case 3: When  $i = n - 1$ ,**

$$f(v_{(n-1)j}) = 3n^2 - \sum_{i=1}^{n-1} (4n - (2i-1)) + 2 + j, 1 \leq j \leq (2n + 1)$$

$$f(v_{(n-1)(2n+2)}) = 2n + 2$$

$$f(v_{(n-1)(2n+3)}) = 2n + 3$$

At  $v_{(n-1)j}$ ,  $1 \leq j \leq 2n$

$$|f(v_{(n-1)j}) - f(v_{(n-1)(j+1)})|$$

$$= \left| \left( 3n^2 - \sum_{i=1}^{n-1} (4n - (2i-1)) + 2 + j \right) - 3n^2 - \sum_{i=1}^{n-1} (4n - (2i-1)) + 2 + j + 1 \right| = 1$$

**When  $j$  is even**

$$|f(v_{(n-1)j}) - f(v_{n(j-1)})| = \left| \left( 3n^2 - \sum_{i=1}^{n-1} (4n - (2i-1)) + 2 + j \right) - 2j \right|$$

$$= \left( 3n^2 - \sum_{i=1}^{n-1} (4n - (2i-1)) + 2 - j \right)$$

**When  $j$  is odd**

$$|f(v_{(n-1)j}) - f(v_{(n-2)(j+1)})|$$

$$= \left| \left( 3n^2 - \sum_{i=1}^{n-1} (4n - (2i-1)) + 2 + j \right) - \left( 3n^2 - \sum_{i=1}^{n-2} (4n - (2i-1)) + 2 + j + 1 \right) \right| = 2n + 2$$

$$|ef_{v_{(n-1)j}}(1) - ef_{v_{(n-1)j}}(0)| = 1$$

**Case 4: When  $i = n$**

$$f(v_{n(2j)}) = 2j - 1, 1 \leq j \leq n$$

$$f(v_{n(2j-1)}) = 2j, 1 \leq j \leq n$$

$$f(v_{n,2n+1}) = 2n + 1$$

**when  $j$  is even**

$$|f(v_{n(2j)}) - f(v_{n(2j-1)})| = |2j - 1 - 2j| = 1$$

$$|f(v_{n(2j)}) - f(v_{n(2j+1)})| = |f(v_{n(2j)}) - f(v_{n(2(j+1)-1)})| = |2j - 1 - 2(j+1)| = 3$$

$$|ef_{v_{n(2j)}}(1) - ef_{v_{n(2j)}}(0)| = 0,$$

**When  $j$  is odd**

$$f(v_{n(2j-1)}) - f(v_{(n-1)2j}) = \left| 2j - \left( 3n^2 - \sum_{i=1}^{n-1} (4n - (2i-1)) + 2 + 2j \right) \right|$$

$$= \left( 3n^2 - \sum_{i=1}^{n-1} (4n - (2i-1)) + 2 \right)$$

$$|ef_{v_{n(2j-1)}}(1) - ef_{v_{n(2j-1)}}(0)| = 1, 1 \leq j \leq n - 1$$

**Case 5: When  $i = -(n - 1)$**

$$f(v_{(-(n-1)j}) = 3n^2 + \sum_{i=1}^{n-2} (4n - (2i-1)) + j, 1 \leq j \leq 2n + 1$$

$$f(v_{(-(n-1)(2n+2)}) = 3n^2 + \sum_{i=1}^{n-2} (4n - (2i-1)) + 2n + 3$$

$$f(v_{(-(n-1)(2n+3)}) = 3n^2 + \sum_{i=1}^{n-2} (4n - (2i-1)) + 2n + 2$$

At  $v_{(-(n-1)j}$ ,  $1 \leq j \leq 2n$

$$|f(v_{(-(n-1)j}) - f(v_{(-(n-1)(j+1)})|$$

$$= \left| \left( 3n^2 + \sum_{i=1}^{n-2} (4n - (2i-1)) + j \right) - \left( 3n^2 + \sum_{i=1}^{n-2} (4n - (2i-1)) + j + 1 \right) \right| = 1$$

**When  $j$  is even**

$$|f(v_{(-(n-1)j}) - f(v_{(-(n-1)(j-1)})|$$

$$= \left| \left( 3n^2 + \sum_{i=1}^{n-2} (4n - (2i-1)) + j \right) - \left( 3n^2 + \sum_{i=1}^{n-1} (4n - (2i-1)) + 1 + j \right) \right| = 2n + 4$$

**When  $j$  is odd**

$$|f(v_{(-(n-1)j}) - f(v_{(-(n-2)(j+1)})|$$

$$= \left| \left( 3n^2 + \sum_{i=1}^{n-2} (4n - (2i-1)) + j \right) - \left( 3n^2 + \sum_{i=1}^{n-3} (4n - (2i-1)) + j + 1 \right) \right| = 2n + 4$$

$$|ef_{v_{-(n-1)j}}(1) - ef_{v_{-(n-1)j}(0)}| = 1$$

**Case 6: When  $i = -n$**

$$f(v_{(-n)2j}) = 3n^2 + \sum_{i=1}^{n-1} (4n - (2i - 1)) + 1 + (2j - 1), 1 \leq j \leq n$$

$$f(v_{(-n)(2j-1)}) = 3n^2 + \sum_{i=1}^{n-1} (4n - (2i - 1)) + 1 + 2j, 1 \leq j \leq n$$

$$f(v_{(-n)(2n+1)}) = 6n^2 - 2n$$

At  $v_{(-n)2j}$

$$\begin{aligned} &|f(v_{(-n)2j}) - f(v_{(-n)(2j-1)})| \\ &= \left| \left( 3n^2 + \sum_{i=1}^{n-1} (4n - (2i - 1)) + 1 + (2j - 1) \right) \right. \\ &\quad \left. - \left( 3n^2 + \sum_{i=1}^{n-1} (4n - (2i - 1)) + 1 + 2j \right) \right| = 1 \end{aligned}$$

$$\begin{aligned} &|f(v_{(-n)2j}) - f(v_{(-n)(2j+1)})| = |f(v_{(-n)2j}) - f(v_{(-n)(2(j+1)-1})| \\ &= \left| \left( 3n^2 + \sum_{i=1}^{n-1} (4n - (2i - 1)) + 1 + (2j - 1) \right) \right. \\ &\quad \left. - \left( 3n^2 + \sum_{i=1}^{n-1} (4n - (2i - 1)) + 1 + 2(j+1) \right) \right| = 3 \end{aligned}$$

$$|ef_{v_{(-n)2j}}(1) - ef_{v_{(-n)2j}(0)}| = 0$$

At  $v_{(-n)(2j-1)}, 2 \leq j \leq n$

$$\begin{aligned} &|f(v_{(-n)(2j-1)}) - f(v_{(-n)2j})| \\ &= \left| \left( 3n^2 + \sum_{i=1}^{n-1} (4n - (2i - 1)) + 1 + 2j \right) \right. \\ &\quad \left. - \left( 3n^2 + \sum_{i=1}^{n-1} (4n - (2i - 1)) + 1 + (2j - 1) \right) \right| = 1 \end{aligned}$$

$$\begin{aligned} &|f(v_{(-n)(2j-1)}) - f(v_{(-n)2(j-1)})| \\ &= \left| \left( 3n^2 + \sum_{i=1}^{n-1} (4n - (2i - 1)) + 1 + 2j \right) \right. \\ &\quad \left. - \left( 3n^2 + \sum_{i=1}^{n-1} (4n - (2i - 1)) + 1 + 2(j-1) - 1 \right) \right| = 3 \end{aligned}$$

$$\begin{aligned} &|f(v_{(-n)(2j-1)}) - f(v_{(-n-1)2j})| \\ &= \left| \left( 3n^2 + \sum_{i=1}^{n-1} (4n - (2i - 1)) + 1 + 2j \right) \right. \\ &\quad \left. - \left( 3n^2 + \sum_{i=1}^{n-2} (4n - (2i - 1)) + 2j \right) \right| = 2n + 4 \end{aligned}$$

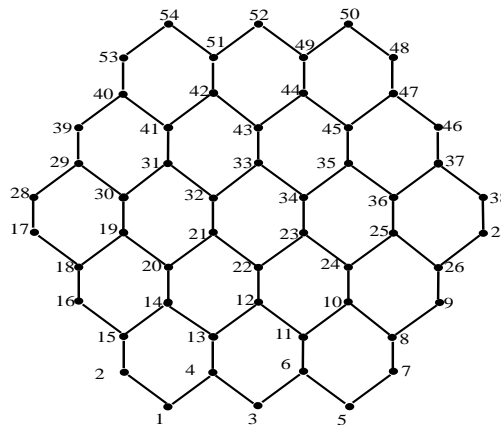


Fig 2: Honeycomb network HC(3)

At each vertex  $v \in HC(n), |ef_v(1) - ef_v(0)| \leq 1$ . Hence any honeycomb network is a neighborhood graph.

**Theorem 2:** Any grid  $M(m \times n)$  is neighbourhood difference cordial graph.

Let  $V = \{v_1, v_2, v_3, \dots, v_{mn}\}$

Define  $f: V \rightarrow \{1, 2, 3, \dots, mn\}$

$$f(v_{ij}) = (i - 1)n + j, 1 \leq i \leq m, 1 \leq j \leq n$$

**Case 1:**

At  $v_{11}, ef_{v_{11}}(1) = 1$  and

$$ef_{v_{11}}(0) = 1. \therefore |ef_{v_{11}}(1) - ef_{v_{11}}(0)| = 0.$$

**Case 2:**

At  $v_{1n}, ef_{v_{1n}}(1) = 1$  and

$$ef_{v_{1n}}(0) = 1. \therefore |ef_{v_{1n}}(1) - ef_{v_{1n}}(0)| = 0.$$

**Case 3:**

At  $v_{m1}, ef_{v_{m1}}(1) = 1$  and

$$ef_{v_{m1}}(0) = 1. \therefore |ef_{v_{m1}}(1) - ef_{v_{m1}}(0)| = 0.$$

**Case 4:**

At  $v_{mn}, ef_{v_{mn}}(1) = 1$  and

$$ef_{v_{mn}}(0) = 1. \therefore |ef_{v_{mn}}(1) - ef_{v_{mn}}(0)| = 0.$$

**Case 5:**

At  $v_{1j}, 2 \leq j \leq n - 1$

$$|f(v_{1j}) - f(v_{1(j+1)})| = |j - (j + 1)| = 1$$

$$|f(v_{1j}) - f(v_{1(j-1)})| = |j - (j - 1)| = 1$$

$$|f(v_{1j}) - f(v_{2j})| = |j - (j + n)| = n$$

At  $v_{1n}, ef_{v_{1j}}(1) = 2$  and

$$ef_{1j}(0) = 1. \therefore |ef_{v_{1j}}(1) - ef_{v_{1j}}(0)| = 1.$$

**Case 6:**

At  $v_{mj}, 2 \leq j \leq n - 1$

$$|f(v_{mj}) - f(v_{m(j+1)})| = |(m - 1)n + j) - (j + 1 + (m - 1)n)| = 1$$

$$|f(v_{mj}) - f(v_{m(j-1)})| = |((m-1)n + j) - (j-1 + (m-1)n)|$$

$$|f(v_{mj}) - f(v_{(m-1)j})| = |((m-1)n + j) - (j + (m-2)n)| = n$$

At  $v_{mj}$ ,  $ef_{v_{mj}}(1) = 2$  and

$$ef_{v_{mj}}(0) = 1 \therefore |ef_{v_{mj}}(1) - ef_{v_{mj}}(0)| = 1$$

Case 7:

At  $v_{i1}$ ,  $2 \leq i \leq m-1$

$$|f(v_{i1}) - f(v_{(i-1)1})| = |((i-1)n + 1) - (1 + (i-2)n)| = n$$

$$|f(v_{i1}) - f(v_{(i+1)1})| = |((i-1)n + 1) - (1 + (i)n)| = n$$

$$|f(v_{i1}) - f(v_{i2})| = |((i-1)n + 1) - (2 + (i-1)n)| = 1$$

At  $v_{i1}$ ,  $ef_{v_{i1}}(1) = 1$  and

$$ef_{v_{i1}}(0) = 2 \therefore |ef_{v_{i1}}(1) - ef_{v_{i1}}(0)| = 1$$

Case 8:

At  $v_{in}$ ,  $2 \leq i \leq m-1$

$$|f(v_{in}) - f(v_{(i-1)n})| = |((i-1)n + n) - (n-1 + (i-1)n)| = 1$$

$$|f(v_{in}) - f(v_{(i+1)n})| = |((i-1)n + n) - (n + (i)n)| = n$$

$$|f(v_{in}) - f(v_{(i-1)(n-1)})| = |((i-1)n + n) - (n + (i-2)n)| = n$$

At  $v_{in}$ ,  $ef_{v_{in}}(1) = 1$  and

$$ef_{v_{in}}(0) = 2 \therefore |ef_{v_{in}}(1) - ef_{v_{in}}(0)| = 1$$

Case 9:

At  $v_{ij}$ ,  $2 \leq i \leq m-1$ ,

$2 \leq j \leq n-1$

$$|f(v_{ij}) - f(v_{i(j-1)})| = |((i-1)n + j) - (j-1 + (i-1)n)| = 1$$

$$|f(v_{ij}) - f(v_{(i+1)j})| = |((i-1)n + j) - (j + (i)n)| = n$$

$$|f(v_{ij}) - f(v_{(i-1)j})| = |((i-1)n + j) - (j + (i-2)n)| = n$$

$$|f(v_{ij}) - f(v_{i(j+1)})| = |((i-1)n + j) - (j+1 + (i-1)n)| = 1$$

At  $v_{ij}$ ,  $ef_{v_{ij}}(1) = 2$  and

$$ef_{v_{ij}}(0) = 2 \therefore |ef_{v_{ij}}(1) - ef_{v_{ij}}(0)| = 0$$

By the above labeling, at each vertex

$v$ ,  $|ef_v(1) - ef_v(0)| \leq 1$ . Hence grid is a neighborhood difference cordial graph.

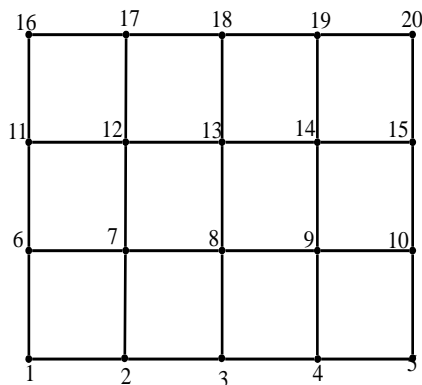


Fig 3: Grid  $M_{4 \times 5}$

**Theorem 3:** Anybutterfly network  $BF(n)$  is neighborhood difference cordial graph.

**Proof:**

The butterfly network has  $(n+1)2^n$  vertices and  $n2^{n+1}$  edges.

Let  $V = \{v_1, v_2, \dots, v_k\}$

Define  $f:V \rightarrow \{1, 2, 3, \dots, (n+1)2^n\}$

$$f(v_{ij}) = (n+1)(j-1) + (i+1), 0 \leq i \leq n, 1 \leq j \leq 2^n.$$

The edge connecting the vertices  $v_{ij}$  and  $v_{(i+1)j}$ ,  $0 \leq i \leq n-1$  are straight edges and other edges are cross edges. By the above labeling the stright edges are labeled with 1 and cross edges are not labeled with 1. At each vertices  $v$ , the no of stright edges and cross edges are equal. Therefore at each vertex  $v \in V(G), |ef_v(1) - ef_v(0)| = 0$ . Hence butterfly network is a neighborhood differential cordial graph.

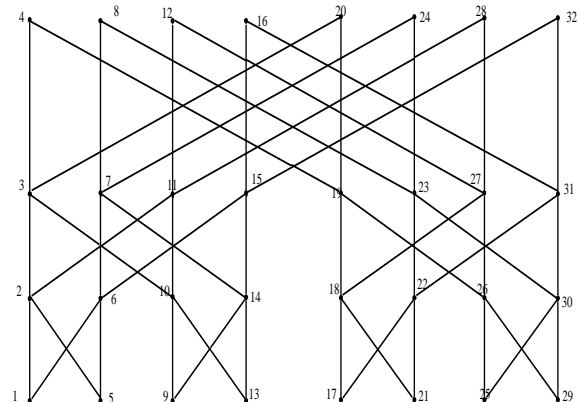


Fig 4: Butterfly network  $BF(3)$

**Theorem 4:** Any benes network is a neighbourhood difference cordial graph.

**Proof:**

The  $n$  dimensional benes network has  $(2n+1)2^n$  vertices and  $n2^{n+2}$  edges. In level  $0$  and level  $n-1$ , vertices are of degree 2 and in other levels, vertices are of degree 4.

Let  $V = \{v_1, v_2, v_3, \dots, v_k\}$

Define  $f:V \rightarrow \{1, 2, 3, \dots, (2n+1)2^n\}$

$$f(v_{ij}) = (2n+1)(j-1) + (i+1), 0 \leq i \leq 2n, 1 \leq j \leq 2^n.$$

**Proof:** The proof is similar to butterfly network.



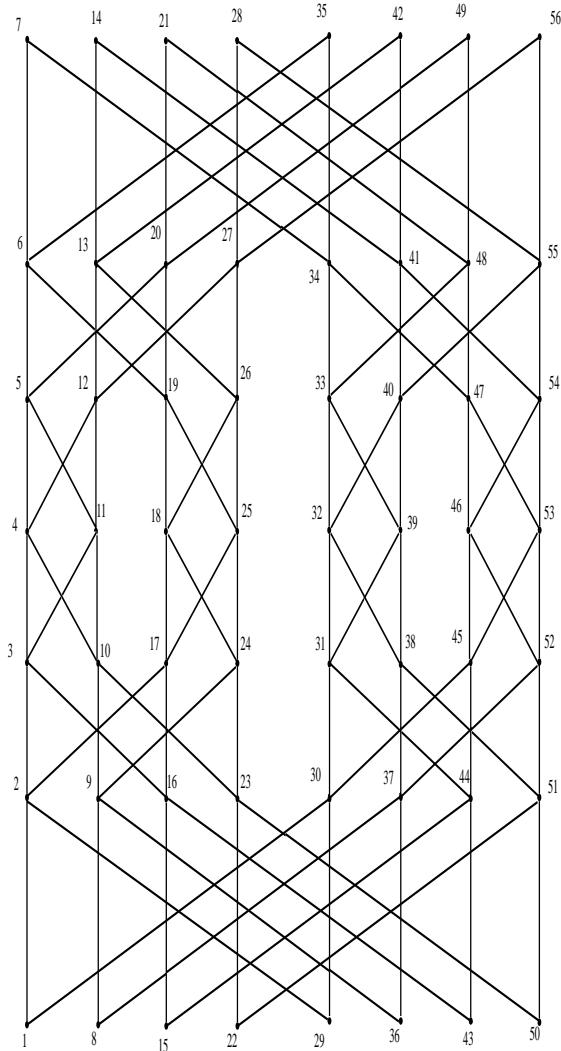


Fig 5: Bene network BB(3)

From the above labeling the bene network is a neighborhood difference cordial graph.

### CONCLUSION

In this paper we proved that honey comb network, grid, butterfly network and bene network are neighbourhood difference cordial graph.

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