

# Existence of Solutions for Random Impulsive Differential Equation with Nonlocal Conditions

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**Abstract**— Impulsive differential equations deals with the study of the dynamic processes which undergo sudden changes. Since 1990's many mathematicians have derived lots of results on differential equations undergoing impulsive effects. Problems including local initial condition and the problems including nonlocal conditions were considered in their work. But the deterministic impulsive differential equations fail to demonstrate many real life situations. And to handle such situations the concept of random impulsive differential equations were introduced. In this paper, we introduce random impulsive differential equations with nonlocal condition. The main aim of this paper to study the existence and uniqueness of solutions of random impulsive differential equations with nonlocal condition. For, we prove a result using fixed point theory technique.

**Keywords**— Existence, Uniqueness, Fixed point theorem, Random impulses, Nonlocal conditions

## I. INTRODUCTION

The study of impulsive differential equations opens a large space for natural frameworks, for many real life mathematical phenomenon's. We can see many researchers dealing with the equations where the systems are allowed to undergo some abrupt perturbations and whose duration can be negligible in comparison with the duration of the process. The differential equations incorporating jump discontinuities for their solutions are called impulsive differential equations. Freedman, Liu and Wu [8] studied models of single species growth with impulsive effect; Zavalishchin [26] studied impulsive dynamic system for mathematical economics. Lakshmikantham, Bainov and Simeonov [14] and Liu [15] also studied more details.

Many real world phenomena lead to sudden changes. While it is not possible to define the system using the uncertainties and complexities related to deterministic equations. And to overcome this crisis a number of stochastic models were proposed. The solutions of the differential equations are stochastic processes if the impulses exist at random points. And it is different from both deterministic impulsive differential equations and stochastic differential equations. Random impulsive equations are more realistic compared to deterministic impulsive equations. And Impulsive differential equations provide a natural and rational approach when operated with random coefficients. There were many publications related to this field. The concept of random

impulsive ordinary differential equations was first introduced by Wu and Meng [21]. They used Liapunov's direct method and found the boundedness of solutions. Iwankiewicz and Nielsen [12], investigated dynamic response of non-linear systems to poisson distributed random impulses. Sanz-Serna and Stuart [18] first brought dissipative differential equations with random impulses and used Markov chains to simulate such systems. K. Tatsuyuki, K. Takashi and S. Satoshi [19] presented a mathematical model of random impulse to depict drift motion of granules in chara cells due to myosin-actin interaction. In [8], the author studied the existence, uniqueness, continuous dependence, Ulam stabilities and exponential stability of random impulsive semi linear differential equations under sufficient condition. Recently Sayooj Aby Jose and Venkatesh Usha [17] investigated the existence and uniqueness of mild solutions for special random impulsive differential equations using contraction mapping principle. This is an area of extensive research activity.

The random impulsive differential equations, describes a system where the initial condition is given by  $u(0) = u_0$ . Differential equations involving non local conditions are more realistic to describe many phenomenon practical than those of local initial conditions. Random impulsive differential equations with non local condition therefore have attracted considerable attention. It is remarkable that, recently many researchers focused on differential equations with non local initial conditions see [ 4, 10, 13, 11 ]. That is,

the classical initial condition (also called "local condition")  $u(0) = u_0$  is extended to the following non-local condition

$$u(0)+g(u) = u_0.$$

where  $u$  is a solution and  $g$  is a mapping defined on some function space into  $X$ . For example, for a non-uniform rod,  $g(u)$  may be given by

$$g(u) = \sum_{i=1}^q C_i u(s_i) \tag{1.1}$$

where  $c_i; i = 1 \dots q;$  are given constants and  $0 < s_1 < s_2 < \dots < s_q$ . In this case (1.1) allows the additional measurements at  $s_i; i = 1, 2, \dots, q$ . A formula similar to (1.1) is also used in Deng [5] to describe the diffusion phenomenon of small amount of gas in a transparent tube. In general,  $g$  may be an integral and may be non-linear. Some more impulsive differential equations with non-local conditions are investigated. If a sound wave travels through a non-uniform rod (where non-local conditions can be applied), and if the wave's amplitude or frequency (parameter) changes in a piecewise continuous fashion with steps, then the vibration in the rod will also contain steps. So the merging of the random impulsive and non-local conditions would be helpful in modelling system. Therefore, we study the existence and uniqueness of solutions for the following random impulsive differential equation with non-local conditions of the form

$$u'(t) = Au(t) + f(t, u(t)), t \neq \xi_k, t \geq \tau$$

$$u(\xi_k) = b_k(\tau_k) u(\xi_k^-), k = 1, 2, 3, \dots,$$

$$u_{t_0} + g(u) = u_0,$$

This paper is organized as follows: Some preliminaries are presented in Section II, followed by some hypotheses in Section III, in IV we investigate the existence and uniqueness of solution of random impulsive differential equation with nonlocal condition. Moreover, Lipschitz condition has to be used for deriving the main results.

**Definition 2.1.** Consider the inhomogeneous initial value problem where  $f : [0, T] \rightarrow X$ .

$$\begin{aligned} u(t) &= Au(t) + f(t) \\ u(0)+g(u) &= u_0. \end{aligned}$$

Let  $A$  be the infinitesimal generator of a  $C_0$  semigroup  $T(t)$ . Let  $u_0 \in X$  and  $f \in L^1([0, T]; X)$ . Then the function  $u \in C([0, T]; X)$  is given by

$$u(t) = T(t)(u_0 - g(u)) + \int_0^t T(t-s)f(s)ds, 0 \leq t \leq T$$

is the mild solution of the above initial value problem for  $t \in [0, T]$ .

**Definition 2.2.** A semigroup  $\{T(t), t \geq 0\}$  is said to be uniformly bounded if there exist a constant  $M \geq 1$  such that

$$\|T(t)\| \leq M, \text{ for } t \geq 0$$

**Definition 2.3.** For a given  $T \in (\tau, +\infty)$ , a stochastic process  $\{u(t) \in \mathcal{B}, \tau \leq t \leq T\}$  is called a mild solution to equation (2.1) – (2.3) in  $(\Omega, P, \{\mathcal{F}_t\})$ , if

## II. PRELIMINARIES

Let  $X$  be a real separable Hilbert space and  $\Omega$  a non empty set. Assume that  $\tau_k$  is a random variable defined from  $\Omega$  to  $D_k = \{(0, d_k)\}$  for all  $k = 1, 2, 3, \dots$  where  $0 < d_k < +\infty$ . Furthermore, assume that  $\tau_i$  and  $\tau_j$  are independent with each other as  $i \neq j$  for  $i, j = 1, 2, \dots$ . Let  $\tau \in \mathcal{R}$  be a constant. For the sake of simplicity, we denote  $\mathcal{R}_\tau = [\tau, T]$ . We consider the differential equations with random impulsive of the form

$$u'(t) = Au(t) + f(t, u(t)), t \neq \xi_k, t \geq \tau \tag{2.1}$$

$$u(\xi_k) = b_k(\tau_k) u(\xi_k^-), k = 1, 2, 3, \dots, \tag{2.2}$$

$$u_{t_0} + g(u) = u_0, \tag{2.3}$$

Where  $A$  is the infinitesimal generator of a strongly continuous semigroup of bounded linear operator  $T(t)$  in  $X$ ;  $f: \mathcal{R}_\tau \times X \rightarrow X, b_k: D_k \rightarrow \mathcal{R}$  for each  $k=1, 2, 3, \dots, \xi_0 = t_0 \in [\tau, T]$  and  $\xi_k = \xi_{k-1} + \tau_k$  for  $k = 1, 2, 3, \dots$ , here  $t_0 \in \mathcal{R}_\tau$  is arbitrary real number. Obviously,  $t_0 = \xi_0 < \xi_1 < \xi_2 < \dots < \xi_k < \dots$ ;  $u(\xi_k^-) = \lim_{t \uparrow \xi_k} u(t)$  according to their path with the norm  $\|u\| = \sup_{\tau \leq t \leq T} |u(s)|$  for each  $t$  satisfying  $\tau \leq t \leq T$

Let us denote  $\{\mathcal{B}_t, t \geq 0\}$  be the simple counting process generated by  $\{\xi_n\}$ , that is,  $\{\mathcal{B}_t \geq n\} = \{\xi_n \leq t\}$ , and denote  $\mathcal{F}_t$  the  $\sigma$ -algebra generated by  $\{\mathcal{B}_t, t \geq 0\}$ . Then  $(\Omega, P, \{\mathcal{F}_t\})$  is a probability space. Let  $L_2 = L_2(\Omega, \{\mathcal{F}_t\}, X)$  denote the Hilbert space of all  $\{\mathcal{F}_t\}$ -measurable square integrable random variables with values in  $X$ .

Let  $\mathcal{B}$  denote Banach space  $\mathcal{B}([\tau, T], L_2)$ , the family of all  $\{\mathcal{F}_t\}$ -measurable random variable  $\psi$  with norm

$$\|\psi\|^2 = \sup_{t_0 \in [\tau, T]} E \|\psi\|^2.$$

- (i)  $u(t) \in X$  is  $\mathcal{F}_t^-$ -adapted;
- (ii)

$$u(t) = \sum_{k=0}^{+\infty} \left[ \prod_{i=1}^k b_i(\tau_i) T(t - t_0)(x_0 - g(u)) + \sum_{i=1}^k \prod_{j=i}^k b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} T(t - s) f(s, u(s)) ds + \int_{\xi_k}^t T(t - s) f(s, u(s)) ds \right] I_{[\xi_k, \xi_{k+1})}(t), \quad t \in [\tau, T] \tag{2.4}$$

where,  $\prod_{j=m}^n (\cdot) = 1$  as  $m > n$ ,  $\prod_{j=i}^k b_j(\tau_j) = b_k(\tau_k) b_{k-1}(\tau_{k-1}) \dots b_i(\tau_i)$ , and  $I_A(\cdot)$  is the index function, i.e

$$I_A(t) = \begin{cases} 1, & \text{if } t \in A \\ 0, & \text{if } t \notin A \end{cases}$$

### III. HYPOTHESES

Before proving the main results, we want some hypotheses which are used in our results.

(H<sub>1</sub>) The function  $f$  satisfies the Lipschitz condition. That is for  $u, v \in X$  and  $\tau \leq t \leq T$ , there exist a constant  $L > 0, G > 0$  such that

$$\begin{aligned} E \| f(t, u) - f(t, v) \| &\leq L E \| u - v \| \\ E \| f(t, 0) \| &\leq k \quad \text{where } k \geq 0 \text{ is a constant} \\ E \| g(u) - g(v) \| &\leq G E \| u - v \| \end{aligned}$$

(H<sub>2</sub>) The condition  $\max_{i,k} \{ \prod_{j=i}^k \| b_j(\tau_j) \| \}$  is uniformly bounded if, there is a constant  $C > 0$  such that

$$\max_{i,k} \left\{ \prod_{j=i}^k \| b_j(\tau_j) \| \right\} \leq C, \quad \text{for all } \tau_j \in D_j, j = 1, 2, 3, \dots$$

(H<sub>3</sub>)

$$\Gamma = M^2 [ GC^2 + \max\{1, C^2\} (T - \tau)^2 L ] < 1 \tag{3.1}$$

### IV. EXISTENCE AND UNIQUENESS

In this section, we discuss the existence and uniqueness of the mild solution for the system (2.1) – (2.3).

**Theorem 3.1.** *Let the hypotheses (H<sub>1</sub>) – (H<sub>3</sub>) holds, then the system (2.1) – (2.3) has a unique mild solution in  $\mathcal{B}$*

*Proof.* Let  $T$  be an arbitrary number  $\tau < T < +\infty$ . First we define the nonlinear operator  $S : \mathcal{B} \rightarrow \mathcal{B}$  as follows

$$Su(t) = \sum_{k=0}^{+\infty} \left[ \prod_{i=1}^k b_i(\tau_i) T(t - t_0)(u_0 - g(u)) + \sum_{i=1}^k \prod_{j=i}^k b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} T(t - s) f(s, u(s)) ds + \int_{\xi_k}^t T(t - s) f(s, u(s)) ds \right]$$

$$+ \int_{\xi_k}^t T(t-s) f(s, u(s)) ds \Big] I_{[\xi_k, \xi_{k+1})}(t), \quad t \in [\tau, T]$$

The continuity of  $S$  can be proved easily. Next we will show that  $\mathcal{B}$  is mapped into  $\mathcal{B}$  under  $S$

$$\begin{aligned} \|Su(t)\|^2 &\leq \left[ \sum_{k=0}^{+\infty} \left[ \left\| \prod_{i=1}^k b_i(\tau_i) \right\| T(t-t_0) \left\| u_0 - g(u) \right\| \right. \right. \\ &\quad \left. \left. + \sum_{i=1}^k \left\| \prod_{j=i}^k b_j(\tau_j) \right\| \left\{ \int_{\xi_{i-1}}^{\xi_i} T(t-s) f(s, u(s)) \right\} \right. \right. \\ &\quad \left. \left. + \int_{\xi_k}^t T(t-s) f(s, u(s)) \right\} I_{[\xi_k, \xi_{k+1})}(t) \right]^2 \\ &\leq 2 \left[ \sum_{k=0}^{+\infty} \left[ \prod_{i=1}^k \|b_i(\tau_i)\|^2 T(t-t_0)^2 \|u_0 - g(u)\|^2 I_{[\xi_k, \xi_{k+1})}(t) \right] \right. \\ &\quad \left. + \left[ \sum_{k=0}^{+\infty} \left[ \sum_{i=1}^k \left\| \prod_{j=i}^k b_j(\tau_j) \right\| \left\{ \int_{\xi_{i-1}}^{\xi_i} T(t-s) \|f(s, u(s))\| ds \right\} \right. \right. \right. \\ &\quad \left. \left. + \int_{\xi_k}^t T(t-s) \|f(s, u(s))\| ds \right\} I_{[\xi_k, \xi_{k+1})}(t) \right]^2 \right] \\ &\leq 2M^2 \max_k \left\{ \left\| \prod_{i=1}^k b_i(\tau_i) \right\|^2 \right\} \|u_0 - g(u)\|^2 \\ &\quad + 2M^2 \left[ \max_{i,k} \left\{ 1, \left\| \prod_{j=i}^k b_j(\tau_j) \right\| \right\} \right]^2 \\ &\quad \times \left( \int_{t_0}^t \|f(s, u(s))\| ds I_{[\xi_k, \xi_{k+1})}(t) \right)^2 \\ &\leq 2M^2 C^2 \|u_0 - g(u)\|^2 + 2M^2 \max\{1, C^2\} \left( \int_{t_0}^t \|f(s, u(s))\| ds \right)^2 \\ &\leq 2M^2 C^2 \|u_0 - g(u)\|^2 + 2M^2 \max\{1, C^2\} (t-t_0) \int_{t_0}^t \|f(s, u(s))\|^2 ds \\ E \|Su(t)\|^2 &\leq 2M^2 C^2 \|u_0 - g(u)\|^2 + 2M^2 \max\{1, C^2\} \\ &\quad \times (T-\tau) \int_{t_0}^t E \|f(s, u(s))\|^2 ds \\ &\leq 2M^2 C^2 \|u_0 - g(u)\|^2 + 4M^2 \max\{1, C^2\} (T-\tau)^2 k \\ &\quad + 4M^2 \max\{1, C^2\} (T-\tau) L \int_{t_0}^t E \|x(s)\|^2 ds \end{aligned}$$

Thus,

$$\sup_{t \in [\tau, T]} E \|Su(t)\|^2 \leq 2M^2 C^2 \|u_0 - g(u)\|^2 + 4M^2 \max\{1, C^2\} (T-\tau)^2 k$$

$$\begin{aligned}
 &+4M^2 \max\{1, C^2\} (T - \tau) L \int_{t_0}^t \text{Sup}_{s \in [\tau, t]} E \|u(s)\|^2 ds \\
 &\leq 2M^2 C^2 \|u_0 - g(u)\|^2 + 4M^2 \max\{1, C^2\} (T - \tau)^2 k \\
 &+4M^2 \max\{1, C^2\} (T - \tau)^2 L \text{Sup}_{t \in [\tau, T]} E \|u(t)\|^2
 \end{aligned}$$

For  $t \in [\tau, T]$ , therefore  $S$  maps  $\mathcal{B}$  into itself.

Now we have to show  $S$  is a contraction mapping

$$\begin{aligned}
 &\|(Su)(t) - (Sv)(t)\|^2 \\
 &\leq \left[ \sum_{k=0}^{+\infty} \left\{ \prod_{i=1}^k b_i(\tau_i) \|T(t - t_0)\| \|g(u) - g(v)\| + \sum_{i=1}^k \prod_{j=i}^k \|b_j(\tau_j)\| \right. \right. \\
 &\quad \times \int_{\xi_{i-1}}^t \|T(t-s)\| \|f(s, u(s)) - f(s, v(s))\| ds \\
 &\quad \left. \left. + \int_{\xi_k}^t \|T(t-s)\| \|f(s, u(s)) - f(s, v(s))\| ds \right\} I_{[\xi_k, \xi_{k+1})}(t) \right]^2 \\
 &\leq M^2 \max_k \left\{ \prod_{i=1}^k b_i(\tau_i) \|T(t - t_0)\| \|g(u) - g(v)\| + M^2 \left[ \max_{i,k} \left\{ 1, \prod_{j=i}^k \|b_j(\tau_j)\| \right\} \right]^2 \right. \\
 &\quad \left. \times \left( \int_{t_0}^t \|f(s, u(s)) - f(s, v(s))\| ds I_{[\xi_k, \xi_{k+1})}(t) \right)^2 \right. \\
 &\leq M^2 C^2 \|g(u) - g(v)\|^2 + M^2 \max\{1, C^2\} (t - t_0) \\
 &\quad \times \left( \int_{t_0}^t \|f(s, u(s)) - f(s, v(s))\|^2 ds \right)
 \end{aligned}$$

Then  $E\|S(u)(t) - S(v)(t)\|^2$

$$\begin{aligned}
 &\leq M^2 C^2 E \|g(u) - g(v)\|^2 + M^2 \max\{1, C^2\} (t - t_0) \int_{t_0}^t E \|f(s, u(s)) - f(s, v(s))\|^2 ds \\
 &\leq M^2 C^2 G E \|x - y\|^2 + M^2 \max\{1, C^2\} (T - \tau) L \int_{t_0}^t E \|x(s) - y(s)\|^2 ds
 \end{aligned}$$

Taking Suprimum over  $t$ , we get,

$$\begin{aligned}
 \|(Su)(t) - (Sv)(t)\|^2 &\leq M^2 C^2 G \|u - v\|^2 + M^2 \max\{1, C^2\} (T - \tau)^2 L \|u - v\|^2 \\
 &\leq M^2 [C^2 G + \max\{1, C^2\} (T - \tau)^2 L] \|u - v\|^2
 \end{aligned}$$

Using (3.1) we get,

$$\|(Su)(t) - (Sv)(t)\|^2 \leq \Gamma \|u - v\|^2$$

since  $0 < \Gamma < 1$ . Thus we get that the operator  $S$  satisfies the contraction mapping principle and it implies,  $S$  has a unique fixed point which is the mild solution of the system (2.1)–(2.3). This completes the proof.

## V. CONCLUSION AND FUTURE SCOPE

In this paper, we have investigated the existence and uniqueness of random impulsive differential equations with nonlocal conditions. Similarly, other kinds of these random impulsive differential equations can be studied by similar techniques and we can extend the obtained results to other field of random impulsive differential equations.

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