IJCSE International Journal of Computer Sciences and Engineering Open Access
Research Paper Vol.-7, Special Issue, 5, March 2019 E-ISSN: 2347-2693

Extension of Binary Topology

R.Seethalakshmi[1*], M.Kamaraj[2]

[1]Deaprtmant of Mathematics, Jaya collage of arts and Science, Thiruninravuir – 602024, Tamilnadu, India
[2]Department of Mathematics, Govt. Arts and Science Collage Sivakasi- 626124, Tamilnadu, India

[*]*Corresponding Author: Seethamaths79@gmail.com*

DOI: https://doi.org/10.26438/ijcse/v7si5.194197 | Available online at: www.ijcseonline.org

Abstract: Nithyanantha Jothi and Thangavelu studied the properties of the product of two power sets and introduced the concept of binary topology. In this paper, properties of the product of arbitrarily n-power sets are discussed where n >2. Further an n-ary topology on the product of power sets similar to binary topology is introduced and studied.

Keywords: Binary topology, n-ary sets, n-ary topology, product topology.
MSC 2010: 54A05, 54A99.

I. INTRODUCTION

The concept of a binary topology was introduced and studied by Nithyanantha Jothi and Thangavelu [4-9] in 2011. Recently Lellish Thivagar et.al.[3] extended this notion to supra topology, Jamal Mustafa[2] to generalized topology and Benchalli et.al.[1] to soft topology. Nithyanantha Jothi and Thangavelu[9] extended the concepts of regular open and semiopen sets in point set topology to binary topology. The authors[10] studied the notion of nearly binary open sets in binary topological spaces. In this paper, the notion of n-ary topology is introduced and its properties are discussed. Section 2 deals with basic properties of the product of power sets and section 3 deals with n-ary topology with sufficient examples and some basic results.

II. PRODUCT OF POWER SETS

Let X, X_1, X_2, X_3,…,X_n be the non empty sets. Then P(X) denotes the collection of all subsets of X, called the power set of X. $P(X_1)\times P(X_2)\times\ldots\times P(X_n)$ is the Cartesian product of the power sets $P(X_1)$, $P(X_2)$,…,$P(X_n)$. Examples can be constructed to show that the two notions 'product of power sets' and 'power set of the products' are different. When $|X_1|=|X_2| =2$, it is noteworthy to see that $|P(X_1\times X_2)|= |P(X_1)\times P(X_2)| =16$. But this is not always true as shown in the next proposition .

Proposition 2.1: Suppose $X_1,X_2,\ldots, X_n$ are finite non empty sets satisfying one of the conditions.
(i).$n>2$, $|X_i|>1$for each $i\in\{1,2,\ldots,n\}$;
(ii).$n=2$, $|X_1|>2$,$|X_2|\geq 2$;
(iii).$n=2$, $|X_2|>2$, $|X_1| \geq 2$.Then
$|P(X_1\times X_2\times\ldots\times X_n)| > |P(X_1)\times P(X_2)\times\ldots\times P(X_n)|$.

Proof: Suppose X_i has m_i elements for each $i\in\{1,2,3,\ldots,n\}$. Then each $P(X_i)$ has 2^{m_i} members. Therefore $X_1\times X_2\times\ldots\times X_n$ has $m_1\times m_2\times\ldots\times m_n$ elements that implies $P(X_1\times X_2\times\ldots\times X_n)$ has $2^{m_1\times m_2\times\ldots\times m_n}$ elements where as $P(X_1)\times P(X_2)\times\ldots\times P(X_n)$ has $2^{m_1}.2^{m_2}\ldots 2^{m_n}$ members. That is $P(X_1)\times P(X_2)\times\ldots\times P(X_n)$ has $2^{m_1+m_2+\ldots+m_n}$ members. Under the conditions on n and $|X_i|$ it follows that $2^{m_1\times m_2\times\ldots\times m_n} > 2^{m_1+m_2+\ldots+m_n}$. This shows that $|P(X_1\times X_2\times\ldots\times X_n)|>|P(X_1)\times P(X_2)\times\ldots\times P(X_n)|$.

Proposition 2.2: Let N be the set of all natural numbers and $a,b,c,d,e\in N$.
(i). The equation ab=a+b has exactly one solution in N.
(ii).The equation abc=a+b+c has at least exactly six solutions in N.

Proof: Solutions can be found by inspection method. If a=1 then ab=a+b gives b=1+b that implies (a=1, b≥1) cannot be a solution for (i). Clearly (a=2, b=1) is not a solution for (i). But (a=2, b=2) is a solution for (i). Now suppose (a,b) is a solution for (i) in N. Then ab=a+b that implies a divides a+b. Since a divides itself it follows that a divides b. Similarly b divides a that implies a=b from which it follows that $a^2=2a$. This proves that a=b=2. Therefore (2,2) is the only solution of (i) in N. This proves (i). Now for the equation (ii), suppose a=1,we get bc=1+b+c. 'b=1' is not possible. If b=2 then 2c=1+1+c=2+c that implies c=2. Therefore (a,b,c) = (1,2,3) is a solution for (ii). The other solutions are (1,3,2), (2,3,1), (2,1,3), (3,1,2), (3,2,1). Hence the equation abc=a+b+c has at least six solutions in N. Now suppose (a,b,c) is any solution in N for (ii). Then no two of a,b,c are equal to 1. For suppose a=b=1. Then from (ii), c= 2+c that is not possible. No two of a,b,c are equal. For if a=b then a^2c = 2a+c that implies c divides 2a. Therefore 2a = kc for some natural number k. Then using this in a^2c = 2a+c we get a^2 = k+1. Therefore 2a = kc=(a^2-1)c which implies

c = 2a/(a^2-1). Since a>1 and since a< (a^2-1 we see that c<2 that implies c=1 . Then using c=1 and a=b in (ii) we get a^2 = 2a+1 which has no solution for a in N. Hence we conclude that a,b,c are all distinct. We assume that a<b<c. If a=1 then bc=1+b+c that implies b=2 and c=3. If a =k>1, b≥k+1 and c ≥k+2 then abc ≥ k(k+1)(k+2)>(k+1)(k+2)>6 and a+b+c ≥ 3k+3 >6. Therefore any solution other than the permutations of (1,2,3) must satisfy a+b+c =abc >6. This shows that the equation (ii) has at least six solutions in N.

Proposition 2.3. For any integer k>2, the equation $a_1+a_2+\ldots+a_k = a_1.a_2.\ldots.a_k$ has at least k(k−1) solutions in N.

Proof: Let $(a_1,a_2,\ldots,a_k)$ be a positive integral solution for the equation

$$a_1+a_2+\ldots+a_k = a_1.a_2.\ldots.a_k. \quad \text{(Eqn.1)}$$

Then it is not possible to have $a_1=a_2=\ldots=a_k=1$. For if $a_1=a_2=\ldots=a_k=1$ then k=1. Again if $a_1=a_2=\ldots=a_{k-1}=1$ and $a_k>1$ then k-1+ a_k = a_k that implies k=1.

Suppose $a_1=a_2=\ldots=a_{k-2}=1$, $a_{k-1}>1$ and $a_k>1$.Then k-2+ $a_{k-1}+a_k = a_{k-1}.a_k$.

Let a_{k-1} =r >1. Then r. a_k = k+r-2 + a_k that implies

$$a_k = \frac{k+r-2}{r-1} = 1+\frac{k-1}{r-1}. \quad \text{(Eqn.2)}$$

If r>k then $\frac{k-1}{r-1}$ is a proper fraction that implies a_k is not an integer. Therefore we have 2≤ r≤ k. If r=2 then a_k =k and if r=k then a_k =k. Therefore if

$a_1=a_2=\ldots=a_{k-2}=1$, $a_{k-1}=2$ and $a_k=k$ then $(a_1,a_2,\ldots,a_k)$ is a solution of (Eqn.1)

This shows that $(a_1,a_2,\ldots,a_k)$ = (1,1,…1,2, k) is a solution for (Eqn.1). Clearly any permutation of (1,1,…1,2, k) is also a solution for (Eqn.1).

Therefore the number of such solutions is $\frac{k!}{(k-2)!}$ = (k−1)k. Depending upon the values of k, (Eqn.1) may have other solutions. For take k=5. Then

The equation abcde = a+b+c+d+e has at least 20 solutions in N which may be got by taking r=2 in (Eqn.2). If we put r =3 in (Eqn.2) we get e=3 that implies (1,1,1,3,3) is also a solution for abcde = a+b+c+d+e. Therefore for k>2, (Eqn.1) has at least k(k-1) solutions in N.

The following proposition can be established by choosing r in Eqn.2 such that $\frac{k-1}{r-1}$ is a positive integer.

Proposition 2.4.

(i).If k=5, 10 then (Eqn.1) has at least 3k(k-1)/2 solutions in N.

(ii).If k=7, 9,11 then (Eqn.1) has at least 2k(k-1) solutions in N.

(iii).If k=13 then (Eqn.1) has at least 3k(k-1) solutions in N.

From the above discussion, the following lemma can be easily established.

Proposition 2.5: For any integer k>1, each of the strict in equations $a_1+a_2+\ldots+a_k < a_1.a_2.\ldots.a_k$ and $a_1+a_2+\ldots+a_k > a_1.a_2.\ldots.a_k$ has at least one solution in N.

The above discussions lead to the following proposition.

Proposition 2.6: Let $|X_i| = m_i$ for each i∈{1,2,3,…,k}. Then

$|P(X_1\times X_2\times\ldots\times X_k)| = |P(X_1)\times P(X_2)\times\ldots\times P(X_k)|$, $P(X_1\times X_2\times\ldots\times X_k)| > |P(X_1)\times P(X_2)\times\ldots\times P(X_k)|$ and $|P(X_1\times X_2\times\ldots\times X_k)| < |P(X_1)\times P(X_2)\times\ldots\times P(X_k)|$ according as $(m_1,m_2,\ldots,m_k)$ is a solution of $a_1+a_2+\ldots+a_k = a_1.a_2.\ldots.a_k$, $a_1+a_2+\ldots+a_k < a_1.a_2.\ldots.a_k$ and $a_1+a_2+\ldots+a_k > a_1.a_2.\ldots.a_k$ respectively.

Any typical element in $P(X_1)\times P(X_2)\times\ldots\times P(X_n)$ is of the form $(A_1,A_2,\ldots,A_n)$ where $A_i\subseteq X_i$ for i∈{1,2,3,…,n}. Suppose $(A_1,A_2,\ldots,A_n)$ and $(B_1,B_2,\ldots,B_n)$ are any two members in $P(X_1)\times P(X_2)\times\ldots\times P(X_n)$. Throughout this chapter we use the following notations and terminologies.

$(X_1, X_2,\ldots,X_n)$ is an n-ary absolute set and (∅, ∅, ∅,…,∅) is an n-ary null set or void set or empty set in $P(X_1)\times P(X_2)\times\ldots\times P(X_n)$.

$(A_1,A_2,\ldots,A_n)\subseteq(B_1, B_2,\ldots,B_n)$ if $A_i\subseteq B_i$ for every i∈{1,2,3,…,n} and

$(A_1,A_2,\ldots,A_n) \neq (B_1, B_2,\ldots,B_n)$ if $A_i\neq B_i$ for some i∈{1,2,3,…,n}. Equivalently $(A_1,A_2,\ldots,A_n)=(B_1, B_2,\ldots,B_n)$ if $A_i=B_i$ for every i∈{1,2,3,…,n}. If $A_i\neq B_i$ for each i∈{1,2,3,…,n} then we say $(A_1,A_2,\ldots,A_n)$ is absolutely not equal to $(B_1, B_2,\ldots,B_n)$ which is denoted as $(A_1,A_2,\ldots,A_n) \,{}^{a}{\neq}\, (B_1, B_2,\ldots,B_n)$. Let $x_i\in X_i$ and $A_i\subseteq X_i$ for every i∈{1,2,3,…,n}. Then $(x_1, x_2,\ldots,x_n)\in(A_1, A_2,\ldots, A_n)$ if $x_i\in A_i$ for every i∈{1,2,3,…,n}.

Definition 2.7: Let X_i be an infinite set for every i∈{1,2,3,…,n}.

$(A_1, A_2, \ldots, A_n)$ is finite if A_i is finite for every i∈{1,2,3,…,n} and is infinite if

A_i is infinite for some i∈{1,2,3,…,n}.

Definition 2.8: Let X_i be an uncountable set for every $i\in\{1, 2, 3,\dots,n\}$,
$(A_1, A_2,\dots,A_n)$ is countable if A_i is countable for every $i\in\{1,2,3,\dots,n\}$ and is uncountable if A_i is uncountable for some $i\in\{1,2,3,\dots,n\}$.

Proposition 2.9: $(x_1, x_2,\dots,x_n)\in(A_1, A_2,\dots,A_n)$ iff $(x_1, x_2,\dots,x_n) \in A_1\times A_2\times\dots\times A_n$.

The notions of n-ary union, n-ary intersection, n-ary complement and n-ary difference of n-ary sets are defined component wise. Two n-ary sets are said to be n-ary disjoint if the sets in the corresponding positions are disjoint and $(A_1, A_2, \dots, A_n)$ is a somewhat empty n-ary set if $A_i \neq\varnothing$ for at least one $i\in\{1,2,3,\dots,n\}$ and $A_j =\varnothing$ for at least one $j\in\{1,2,3,\dots,n\}$.

Let S denote the collection of all somewhat empty n-ary sets in $P(X_1)\times P(X_2) \times\dots\times P(X_n)$. Let $M=P(X_1)\times P(X_2) \times\dots\times P(X_n) \setminus S$, the collection of all n-ary sets other than somewhat empty n-ary sets. The next proposition shows that M can be considered as a proper subset of $P(X_1\times X_2\times\dots\times X_n)$.

Proposition 2.10: Let $\varphi: M \to P(X_1\times X_2\times\dots\times X_n)$ be defined by $\varphi(A_1,A_2,\dots,A_n) =A_1\times A_2\times\dots\times A_n$ for each element $(A_1,A_2,\dots,A_n)$ in $P(X_1)\times P(X_2)\times\dots\times P(X_n)$. Then φ is injective but not surjective.
Proof: Suppose $(A_1, A_2,\dots,A_n)$ and $(B_1, B_2,\dots,B_n)$ are any two distinct members of M. Then $A_i\neq B_i$ for some $i\in\{1,2,3,\dots,n\}$ that implies $A_1\times A_2\times\dots\times A_n \neq B_1\times B_2\times\dots\times B_n$. Therefore
$\varphi(A_1, A_2,\dots,A_n) \neq \varphi(B_1, B_2,\dots,B_n)$ that implies φ is injective. Further φ is not surjective as shown below.
$X_1=\{a, b, c\}$ and $X_2=\{1, 2\}$, $A_1\subseteq X_1$ and $A_2\subseteq X_2$. Let $S=\{(a, 1), (b, 2)\}\subseteq X_1\times X_2$. It can be seen that there is no $(A_1, A_2)\in P(X_1\times X_2)$ such that $A_1\times A_2=S$.

Remark 2.11: The function φ, defined above is not injective if we replace M by $P(X_1)\times P(X_2)\times\dots\times P(X_n)$.

Let $f:Y\to X_1\times X_2\times\dots\times X_n$ be a single valued function. Then it induces a function
$f^{-1}:P(X_1)\times P(X_2)\times\dots\times P(X_n)\to P(Y)$ that is an n-ary set to set valued function defined by $f^{-1}((A_1, A_2,\dots,A_n))=\{y:f(y)\in(A_1, A_2,\dots,A_n)\}=\{y: p_i(f(y))\in A_i$ for each $i\in\{1,2,3,\dots,n\}\}$ where each p_i is a projection of $X_1\times X_2\times\dots\times X_n$ onto X_i .

Proposition 2.12: Let $f:Y\to X_1\times X_2\times\dots\times X_n$ be a single valued function. Then
$f^{-1}((A_1, A_2,\dots,A_n))= f^{-1}(A_1\times A_2\times\dots\times A_n)$ for every $(A_1, A_2,\dots,A_n) \in M$.
Proof: $f^{-1}((A_1, A_2,\dots, A_n)) =\{y: f(y)\in(A_1, A_2,\dots,A_n)\}$
$=\{y: p_i(f(y))\in A_i$ for each $i\in\{1,2,3,\dots,n\} \} = \{y: f(y)\in A_1\times A_2\times\dots\times A_n\}$
$= f^{-1}(A_1\times A_2\times\dots\times A_n)$.

Proposition 2.13: Let $f:Y\to X_1\times X_2\times\dots\times X_n$ be a single valued function. Then f^{-1} preserves n-ary union and n-ary intersection.

Proposition 2.14: $P(X_1)\times P(X_2)\times\dots\times P(X_n)$ is a complete distributive lattice under n-ary set inclusion relation.

III. n-ary topology

Let $X_1, X_2, X_3,\dots,X_n$ be the non empty sets. Let $T\subseteq P(X_1)\times P(X_2)\times\dots\times P(X_n)$.
Definition 3.1: T is an n-ary topology on $(X_1, X_2, X_3, \dots, X_n)$ if the following axioms are satisfied.
(i). $(\varnothing, \varnothing, \varnothing,\dots,\varnothing)\in T$
(ii). $(X_1, X_2, X_3, \dots, X_n)\in T$
(iii) If $(A_1, A_2,\dots, A_n), (B_1, B_2, \dots, B_n)\in T$ then
$(A_1, A_2,\dots, A_n)\cap(B_1, B_2,\dots,B_n)\in T$
(iv) If $(A_{1\alpha}, A_{2\alpha}, \dots, A_{n\alpha})\in T$ for each $\alpha\in\Omega$ then $\bigcup_{\alpha\in\Omega}(A_{1\alpha}, A_{2\alpha},\dots, A_{n\alpha}) \in T$.

If T is an n-ary topology then the n+1 tuple $(X_1, X_2, X_3,\dots,X_n ; T)$ is called an n-ary topological space. The elements $(x_1,x_2,\dots,x_n)\in X_1\times X_2\times\dots\times X_n$ are called the n-ary points of $(X_1, X_2, X_3,\dots,X_n;T)$ and the members $(A_1,A_2,\dots,A_n)$ of $P(X_1)\times P(X_2)\times\dots\times P(X_n)$. are called the n-ary sets of $(X_1, X_2,\dots,X_n; T)$. The members of T are called the n-ary open sets in $(X_1, X_2,\dots,X_n ; T)$. If $X_1=X_2=X_3 =\dots =X_n=X$ then T is an n-ary topology on X and (X, T) is an n-ary topological space. It is noteworthy to see that product topology on $X_1\times X_2\times\dots\times X_n$ and n-ary topology on $(X_1, X_2, X_3, \dots, X_n)$ are independent concepts as any open set in product topology is a subset of $X_1\times X_2\times X_3\times\dots\times X_n$ and an open set in an n-ary topology is a member of $P(X_1)\times P(X_2)\times\dots\times P(X_n)$.

Examples 3.2:
(i).I = { $(\varnothing, \varnothing, \varnothing,\dots,\varnothing), (X_1, X_2, X_3, \dots, X_n)$} is an n-ary topology, called indiscrete n-ary topology .
(ii). $D = P(X_1)\times P(X_2)\times\dots\times P(X_n)$ is an n-ary topology, called discrete n-ary topology .
(iii). $\{(\varnothing, \varnothing, \varnothing,\dots,\varnothing)\}\cup\{ (A_1, A_2, A_3, \dots, A_n) : (a_1, a_2, a_3, \dots, a_n)\in (A_1, A_2, A_3, \dots, A_n) \}$ is an n-ary topology, called n-ary point inclusion n-ary topology.
(iv). $\{(\varnothing, \varnothing, \varnothing,\dots,\varnothing)\}\cup\{ (A_1, A_2, A_3, \dots, A_n) : (B_1, B_2, B_3, \dots, B_n) \subseteq (A_1, A_2, A_3, \dots, A_n) \}$ is an n-ary topology, called n-ary set inclusion n-ary topology.
(v). $\{ (\varnothing, \varnothing, \varnothing,\dots,\varnothing), (A_1, A_2, A_3, \dots, A_n) , (X\setminus A_1, X\setminus A_2, X\setminus A_3, \dots,X\setminus A_n), (X_1, X_2, X_3, \dots, X_n) \}$ is an n-ary topology .

(vi). If $(B_1, B_2, B_3, \ldots, B_n) \subseteq (A_1, A_2, A_3, \ldots, A_n)$ then $\{(\emptyset, \emptyset, \emptyset,\ldots,\emptyset), (B_1, B_2, B_3, \ldots, B_n), (A_1, A_2, A_3, \ldots, A_n), (X_1, X_2, X_3, \ldots, X_n)\}$ is an n-ary topology.

(vii). If $(A_1, A_2,\ldots,A_n)$ then $\{(\emptyset, \emptyset, \emptyset,\ldots,\emptyset), (A_1, A_2, A_3, \ldots, A_n), (X_1, X_2, X_3, \ldots, X_n)\}$ is an n-ary topology.

(viii). $Diag(X) = \{\{ (A_1, A_1,\ldots,A_1) : A_1 \subseteq X \}$ is an n-ary topology .

(ix). $F = \{(\emptyset, \emptyset, \emptyset,\ldots,\emptyset)\} \cup \{ (A_1, A_2,\ldots,A_n) : (X\backslash A_1, X\backslash A_2, X\backslash A_3, \ldots, X\backslash A_n)$, is finite } is an n-ary topology called co-finite n-ary topology.

(x). $C = \{(\emptyset, \emptyset, \emptyset,\ldots,\emptyset)\} \cup \{ (A_1, A_2,\ldots,A_n) : (X\backslash A_1, X\backslash A_2, X\backslash A_3, \ldots, X\backslash A_n)$, is countable } is an n-ary topology called co-countable n-ary topology.

(xi). $\{(X, X, X, \ldots,X)\} \cup \{ (A_1, A_2, A_3, \ldots, A_n) : (a_1, a_2, a_3, \ldots, a_n) \notin (A_1, A_2, A_3, \ldots, A_n)\}$ is an n-ary topology, called n-ary point exclusion n-ary topology.

(xii). Let R= the set of all real numbers and R^n = the Cartesian product of n-copies of R. Let $a=(a_1, a_2, \ldots, a_n) \in (R, R,\ldots, R)$. Then for each $r=(r_1, r_2, \ldots, r_n) > 0$. Let $S(a, r)=(S(a_i, r_i),\ldots, S(a_n, r_n))$ where $S(a_i, r_i)=\{x_i: |x_i-a_i|<r_i\}$ for $i \in \{1,2,3,..,n\}$. Let E = the set of all n-ary sets $(A_1, A_2,\ldots, A_n) \in P(R)\times P(R)\times\ldots\times P(R)$ such that for every $(a_1, a_2,\ldots,a_n) \in (A_1, A_2,\ldots,A_n)$ there exists $(r_1, r_2,\ldots, r_n) > 0$ such that
$S(a_i, r_i) \subseteq A_i$ for $i \in \{1,2,3,..,n\}$. Then E is an n-ary topology, called n-ary Euclidean topology over R.

The next proposition is easy to establish.

Proposition 3.3: Let $f:Y \rightarrow X_1\times X_2\times\ldots\times X_n$ be a single valued function. Let T be an n-ary topology on $(X_1, X_2,\ldots,X_n)$. Then $f^{-1}(T) = \{f^{-1}((A_1, A_2, A_3, \ldots, A_n)): (A_1, A_2, A_3, \ldots, A_n) \in T\}$ is a topology on Y.

Proposition 3.4: Let $f_i:Y \rightarrow X_i$ be a single valued function for every $i \in \{1,2,3,\ldots,n\}$. Let $f = (f_1, f_2,\ldots,f_n):Y \rightarrow X_1\times X_2\times\ldots\times X_n$ be defined by $f(y) = (f_1(y), f_2(y),\ldots,f_n(y))$ for every $y \in Y$ where $f_i(y) = p_i(f(y))$ for every $y \in Y$. Let τ be a topology on Y. Then if f is a bijection then $\{f(A): A \in \tau\}$ is an n-ary topology on $(X_1, X_2,\ldots,X_n)$.

Proof: Let A be a subset of Y and $y \in A$ with $f(y) \in f(A)$.Then
$f(y) = (f_1(y), f_2(y),\ldots,f_n(y)) \in (f_1(A), f_2(A),\ldots,f_n(A))$. Conversely let
$(x_1, x_2,\ldots,x_n) \in (f_1(A), f_2(A),\ldots,f_n(A))$ that implies $x_i=f_i(y)=p_i(f(y))$ for some $y \in A$, $i \in \{1,2,3,\ldots,n\}$. That is $(x_1, x_2,\ldots,x_n) \in f(A)$. Therefore $f(A)=(f_1(A), f_2(A),\ldots,f_n(A)) \in P(X_1)\times P(X_2)\times\ldots\times P(X_n)$. Since f is a bijection, each f_i is also a bijection. Therefore it is easy to verify that $\{f(A):A \in \tau\}$is an n-ary topology on $(X_1, X_2,\ldots,X_n)$.

Proposition 3.5: Let p be a permutation of (1,2,,…,n) defined by $p(i)=p_i$, $i \in \{1,2,3,\ldots,n\}$. Let T be an n-ary topology on $(X_1, X_2,\ldots,X_n)$. $p(T) = \{(A_{p(1)}, A_{p(2)},\ldots,A_{p(n)}): (A_1, A_2,\ldots,A_2) \in T\}$ is an n-ary topology on $(X_{p(1)}, X_{p(2)},\ldots,X_{p(n)})$.

Proof: Follows from the fact that $(A_{p(1)}, A_{p(2)},\ldots,A_{p(n)}) \in P(X_{p(1)})\times P(X_{p(2)}) \times\ldots\times P(X_{p(n)})$ iff $(A_1, A_2,\ldots,A_2) \in P(X_1)\times P(X_2)\times\ldots\times P(X_n)$.

The next two propositions can be proved easily.

Proposition 3.6: Let T be an n-ary topology on $(X_1, X_2,\ldots,X_n)$. $T_1 = \{A_1: (A_1, A_2,\ldots,A_2) \in T\}$ and T_2 , T_3 , …, T_n can be similarly defined. Then T_1 , T_2 ,, …, T_n are topologies on $X_1, X_2, \ldots,X_n$ respectively.

Proposition 3.7: Let $\tau_1, \tau_2, \ldots,\tau_n$ be the topologies on $X_1, X_2, \ldots,X_n$ respectively. Let $T = \tau_1\times\tau_2 \times\ldots\times\tau_n = \{(A_1, A_2,\ldots,A_2): A_i \in \tau_i \}$. Then T is an n-ary topology on $(X_1, X_2,\ldots,X_n)$. Moreover $T_i = \tau_i$ for every $i \in \{1,2,3,\ldots,n\}$.

IV. CONCLUSION

The concept of binary topology has been extended to n-ary topology for n>1 sets. The basic properties have been discussed. In particular it is observed that the notions of product topology and n-ary topology are different. More over the connection between the product topology and an n-ary topology is studied.

REFERENCES

[1] S.S.Benchalli , P.G.Patil, S.Abeda Dodamani and J.Pradeepkumar, "On Binary Soft Separation Axioms in Binary Soft Topological Spaces", *Global Journal of Pure and Applied Mathematics*, vol.13, no.9, pp. 5393-5412, 2017.

[2] M.Jamal Mustafa, "On Binary Generalized Topological Spaces", *General Letters in Mathematics*, vol.2, no.3, pp.111-116, 2017.

[3] M.Lellish Thivagar, J. Kavitha, "On Binary Structure of Supra topological spaces", *Bol.Soc. Paran. Mat.,* vol.35, no.3, pp.25-37, 2017.

[4] S.Nithyanantha Jothi, "Binary semicontinuous functions", *International Journal of Mathematics Trends and Technology*, vol.49, no.2, pp.152-155, 2017.

[5] S.Nithyanantha Jothi , P.Thangavelu, "Topology Between Two Sets", *Journal of Mathematical Sciences and Computer Applications*, vol.1, no.3, pp.95-107, 2011.

[6] S.Nithyanantha Jothi , P.Thangavelu, "On Binary Topological Spaces", *Pacific-Asian Journal Mathematics*, vol.5, no.2, pp.133-138, 2011.

[7] S.Nithyanantha Jothi, P.Thangavelu, "Generalized Binary Regular Closed Sets", *IRA International Journal of Applied Sciences*, vol.4, no.2, pp.259-263, 2011.

[8] S.Nithyanantha Jothi, P.Thangavelu , "On Binary Continuity and Binary Separation Axioms", *Ultra Scientists* vol.24, no.1A, pp.121-126, 2012.

[9] S.Nithyanantha Jothi, P.Thangavelu, "Binary semiopen sets in Binary Topological Spaces", *International Journal of Mathematical Archiv*, vol.7, no.9, pp.73-76, 2016.

[10] R.Seethalakshmi, K.Kamaraj, "Nearly Binary Open Sets in Binary Topologica; Spaces", International Conference On Applied and Computational Mathematics(ICACM- 2018),
Erode Arts and Science College(Autonomous) Rangampalaym, Erode-638009, Tamilnadu, India, 2018.