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On the Doubly Edge Geodetic Number of a Graph

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Abstract**—** Geodetic number and its variants is one of the widely studied topic in the field of graph theory. Over the recent years many variants of geodetic number have been extensively studied in the literature. In this paper, we introduce a new variation called doubly edge geodetic number and proved that it is Np-complete. The doubly edge geodetic number for some standard graphs is determined. Furthermore, certain characterization and realization results of doubly edge geodetic number are discussed.

Keywords**—**doubly edge geodetic set, doubly geodetic set, geodesic, geodetic set.

I. INTRODUCTION

In recent times, geodetic problems has become the key interest among the scientists and researcher. In particular, the edge geodetic number has a wide application in many fields like routing, transportation, networking and many more. Many variants of this parameter are defined and been extensively studied by researchers. Most of these variants are derived from the edge geodetic problem by adding some extra constraints to it. But some of the variants of this parameter are defined from its corresponding geodetic problems involving the vertices. In we had introduced a geodetic variant namely doubly geodetic number and later shown this to be NP-complete. In this paper, we define doubly edge geodetic number of a graph and analyze certain properties and computational complexity of this parameter. In the subsequent sections, we formally define and introduce the concept of doubly edge geodetic number of a graph and prove that the problem is NP-complete. We determine the doubly edge geodetic number for certain standard graphs. Also, we obtain certain bounds, characterization and realization results.

II. DOUBLY EDGE GEODETIC NUMBER

Let $G = (V(G), E(G))$,be a non-trivial connected graph. Let $g_p(x, y)$ and $g_q(u, v)$ denote a $x - y$ geodesic and $u - v$ geodesic respectively for any arbitary vertices x, y, u, v in G. We say these two geodesic are distinct if $I[g_p(x, y)] \neq I[g_q(u, v)]$.

A set S of vertices of G is called a *doubly edge geodetic set* of G if each edge $e \in E(G)/E(S)$ lies on at least two distinct geodesics of vertices of S, where $E(S)$ is the edge set of the sub graph induce by the vertices of S. The doubly edge geodetic number of $\ddot{d}g_e(G)$ is minimum cardinality of a doubly edge geodetic set. Any doubly edge geodetic set of cardinality $\ddot{d}g_e(G)$ is called $\ddot{d}g_e$-set of G.

For the graph G_1in Figure 1 (a), it is clear that $u, v$is a doubly edge geodetic set. For the graph G_2 in Figure 1 (b), it is clear that no 2-element or no 3-element subset of G_2 is a doubly edge geodetic set. The set $\{v_1, v_2, v_3, v_5\}$ is a doubly edge geodetic set.

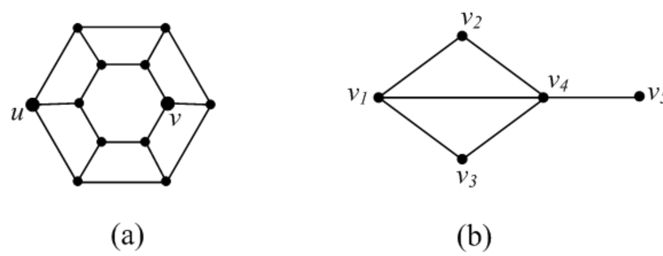

Figure 1: Illustration of Doubly Edge Geodetic number of a Graph

III. COMPUTATIONAL COMPLEXITY

In this section we prove that the doubly edge geodetic problem is NP complete. The proof is given by reduction from a well know NP-problem namely, the vertex cover.

VERTEX COVER

Instance: An undirected graph G and a positive integer k.
Question: Is there a vertex cover of size k or less for G, i.e., a subset V' of V with the size of V' less than K such that every edge has at least one endpoint in V'.

DOUBLY EDGE GEODETIC SET
Instance: A nontrivial connected graph G and a positive integer k.
Question: Is there a doubly edge geodetic set of G set with cardinality k or less ?

Theorem 1 *The doubly edge geodetic set problem is NP-complete.*
Proof. Let $G(V,E)$ be the given graph. The graph $G'(V',E')$ can be constructed from $G(V,E)$ as follows. The vertex set is $V' = x \cup V \cup V' \cup V''$, where the set V' induces a clique and V'' is an independent set of pendant vertices of order $2|V|$. The edge set $E' = E \cup E' \cup E'' \cup E'''$. The vertex set V' along with the edge set E' form a complete graph. the edge set E'' is given by $E'' = \{(v',(v_1)'') \cup (v',(v_2)'')\}$ and $E''' = \{(x,v)\}$.Let S be a vertex cover set of G then clearly $S \cup V'' \cup x$ is a doubly edge geodetic set of G'. Conversely , let $S \cup V'' \cup x$ be a doubly edge geodetic set of G'. By construction distance between any two vertices of V in G' is two. Let $e = uv$ be any edge of G then since $S \cup V'' \cup x$ is a doubly edge geodetic set of G', e will lie in at least two geodesics. Hence any edge will have atleast one end point in S. Hence S is a vertex cover of G.

IV. DOUBLY EDGE GEODETIC NUMBER OF CERTAIN STANDARD GRAPHS

Theorem 2 *For any graph H of order k, $2 \leq g_e(H) \leq \ddot{d}g_e(H) \leq k$ and the bounds are sharp.*

Proof. A edge geodetic set needs at least two vertices and therefore $g_e(H) \geq 2$. It is clear that every doubly edge geodetic set is also a geodetic set and so $g_e(H) \leq \ddot{d}g_e(H)$. Since the set of all vertices of H is a doubly edge geodetic set of $H, \ddot{d}g_e(H) \leq k$.

Remark 1 *The bounds are sharp. For the complete graph $K_n, (n \geq 2)$, we have $\ddot{d}g_e(K_n) = n$and for the path of length 2, $\ddot{d}g_e(P_2) = 2$. The graphs with double edge geodetic number 2 are investigated in the sequel.*

Theorem 3 *Every $\ddot{d}g_e(H)$-set of a graph H contains its extreme vertices.*

Proof. Since every doubly edge geodetic set is a edege geodetic set, the result follows from fact that every edge geodetic set contains its exterme vertices.

Theorem 4 *Let H be a connected graph with a cut vertex u. Then each doubly edge geodetic set contains at least one vertex from each component of $H - u$.*

Proof. This follows the fact that every doubly edge geodetic set is a edge geodetic set and each edge geodetic set contains at least one vertex from each component of $H - u$.

Theorem 5 *If H is a non-trivial connected graph of order k and diameter d, then $\ddot{d}g_e(H) \leq k - d + 2$.*

Proof. Let x and y be vertices of H such that $diam(x,y) = d$. Suppose $P: x = y_0, y_1, \dots, y_d = y$ is a xy -diametral geodesic . Then it is clear that the set $S = V - \{y_1, y_2 \dots, y_{d-2}\}$ is a doubly edge geodetic set of H. Thus $\ddot{d}g_e(H) \leq k - d + 2$.

Theorem 6 *For a tree T with order n and $l (l \geq 3)$ leaves, $\ddot{d}g_e(T) = g(T) = l$.*

Proof. Let L be the set of all end-vertices of T. Clearly, $\ddot{d}\, g_e(T) \geq L$. On the other hand, for an internal edge e of T, there exist end-vertices x, y of T such that e lies on the unique $x - y$ geodesic in T. Thus, an internal edge e of T, will lie on exactly $\binom{l}{2}$ distinct geodesics of vertices in L. Therefore, $d = |L|$. Since every geodetic set L of T must contain L , L is the unique minimum doubly edge geodetic set.

Theorem 8 *The doubly edge geodetic number for some standard graphs:*
(i) $\ddot{d}g_e(P_n) = 3$.
(ii) $\ddot{d}g_e(C_n) = \begin{cases} 4 \text{ if } n \text{ is even} \\ 5 \text{ if } n \text{ is odd} \end{cases}$
(iii) $\ddot{d}g_e(K_{1,n}) = n, n > 2$.
(iv) $\ddot{d}g_e(K_{m,n}) = min\{m,n\}, m, n > 2$.
(v) $\ddot{d}g_e(W_n) = n - 1, n > 4$

Theorem 9 *Let H be a block graph with n vertices and S be the set of all extreme vertices of H then $\ddot{d}g_e(H) = |S|$*

Proof. Let S be the set of all extreme vertices of a block graph H. Its easily verivied that S is a doubly edge geodetic set. Hence the proof.

Theorem 10 *For a n-dimensional hexagonal silicate network $SL(n)$ with k extreme vertices, $\ddot{d}g_e(SL(n)) = k$.*

Proof. Let S be the set of all extreme vertices of a a n-dimensional hexagonal silicate network $SL(n)$. Its easily verivied that S is a doubly edge geodetic set. Hence the proof.

Theorem 11 *If every non end vertex of a tree T is adjacent to at least one end vertex, then $\ddot{d}g_e[L(T)] \leq \left\lceil n - \frac{l}{2} \right\rceil$, where l is number of leaves in T.*

Proof. If $diam(T) \leq 3$, then the result is obvious. Let $diam(T) \geq 3$ and S be the set of all end vertices in T with cardinality k. Now without loss of generality, every end edge of T are the extreme vertices of $L(T)$. Clearly in all the cases we observe that $\ddot{d}g_e[L(T)] \leq \left\lceil n - \frac{l}{2} \right\rceil$, where l is number of leaves in T.

Theorem 12 *Let H be the graph obtained by adding end edge* $\{u_i v_i\}, i = 1,2 \dots n$ *to each vertex of* C_n *such that* $u_i \in C_n, v_i \notin C_n$. *Then* $\ddot{d}g_e(L(G)) = n$.

Proof. Let $\{e_1, e_2, .. e_n\}$ be a cycle with n vertices which is even and let G' be the graph obtained by adding end edge $u_i v_i, i = 1,2 \dots n$ to each vertex of C_n such that $u_i \in C_n, v_i \notin C_n$.. By the definition of line graph, $L(G')$ has K_3 as an induced sub graph. Also the edge $u_i v_i, i = 1,2 \dots n$ becomes a vertex of $L(G')$ and it belongs to some edge geodetic set of $L(G')$. Therefore $\ddot{d}g_e[L(G)] = n$.

Theorem 14 *For the grid* $G_{(r,s)}, \ddot{d}g_e(G_{(r,s)}) = 4$, *where integers* $r, s \geq 2$.

Proof. Let $G = G_{(r,s)}, r, s \geq 2$ and let S be a $\ddot{d}g_e(G)$ -set of G. Suppose S is a 3-element subset of $V(G)$. Then there exist a vertex in the outer boundary, that is the outer cycle $C_{(2(r+s-2))}$,such that it lies on at most one geodesic of vertices of S.Therefore, $\ddot{d}g_e(G) \geq 4$. It can be easily verified that, $S = \{a, b, c, d\}$, where a, b, c, d are the corner vertices is a doubly edge geodetic set of G. Thus, it follows that $\ddot{d}g_e(G) = 4$.

Theorem 15 *Let G be a 3-dimensional grid, then* $\ddot{d}g_e(G) = 2$.

Proof. Let $G = G_{(r,s,t)}, r, s, t \geq 2$ and let S be a $\ddot{d}g_e(G)$ -set of G. Clearly $|S| \geq 2$. Suppose S is a two element set such that the two dimetrically opposite conner vertices are chosen. Then it can be easily verified that, S, is a doubly edge geodetic set of G. Thus, it follows that $\ddot{d}g_e(G) = 2$

Theorem 16 *Let G and H be connected graphs. Then*

$$\ddot{d}g_e(G \square H) \geq max\{g_e(G), g_e(H)\}.$$

Proof. Claim: Let S be a doubly edge geodetic set of $G \square H$. Then the projections $\Pi_G(S)$ and $\Pi_H(S)$ are doubly edge geodetic sets of G and H respectively. Let $e = xy$ be an edge in G, then $e_y = (x, y)(x', y)$ is an edge in $G \square H$ for each $y \in H$. Since S is an doubly edge geodetic set of $G \square H$, e_y lies on at least two distinct geodesics say $P_1: (g_1, h_1) - (g_2, h_2)$; $P_2: (g_3, h_3) - (g_4, h_4)$. Let $\Pi_G(P_1)$ be the projection of P on G then its a $g_1 - g_2$ geodesic in G and $e = xx'$ lies on $\Pi_G(P_1)$. Similarly, $\Pi_G(P_2)$ its a $g_3 - g_4$ geodesic in G and $e = xx'$ lies on $\Pi_G(P_2)$. Hence, $\Pi_G(S)$ is a doubly edge geodetic set of G. Similarly,$\Pi_H(S)$ is a doubly edge geodetic set of H. Now let S be a minimum doubly edge geodetic set of $G \square H$. Then $\ddot{d}g_e(G \square H) = |S|$.
Also since the projections $\Pi_G(S)$ and $\Pi_H(S)$ are doubly edge geodeti sets of G and H respectively we have, $\ddot{d}g_e(G) \leq |\Pi_G(S)|$ and $\ddot{d}g_e(H) \leq |\Pi_H(S)|$, implying $|S| \geq max\{g_e(G), g_e(H)\}$. Hence the theorem.

V. REALIZATION RESULTS OF DOUBLY EDGE GEODETIC SET

Theorem 17 *For positive integers n and k such that* $3 \leq k < n$. *There exist a graph G of order n and* $g(G) = \ddot{d}g_e(G) = k$.

Proof. For $k = n$, let $G = K_n$. Then, $g(G) = \ddot{d}g_e(G) = n = k$. Also, for each pair of integers k, n with $3 \leq k < n$, there exists a tree of order n with k end vertices. Hence the result.

Theorem 18 *For positive integers* r, d *and* $k \geq 3$ *with* $r < d \leq 2r$, *there exists a graph G with* $rad(G) = r$, $diam(G) = d$, $g(G) = \ddot{d}g_e(G) = k$.

Proof. When $r = 1$ and $d = 1$, we have $G = K_k$ with $g(G) = \ddot{d}g_e(G) = k$. When $r = 1$ and $d = 2$, we have $G = K_{1,k}$ with $g(G) = \ddot{d}g_e(G) = k$. For $r \geq 2$ we construct a graph G with desired properties as follows. Let $C_2r: v_1, v_2, \dots, v_{2r}, v_1$ be a cycle of order $2r$ and let $P_{(d-r+1)}: u_0, u_1, u_2, \dots, u_{(d-r)}$ be a path of order $d - r + 1$. Let H be a graph obtained form C_{2r} and $P_{(d-r+1)}$ by identifying v_1 in C_{2r} and u_0 in $P_{(d-r+1)}$. Now, add $k - 2$new vertices $w_1, w_2, w_3, \dots, w(k-2)$ to H by joining each vertices $w_i, 1 \leq i \leq k - 2$ to the vertex $u_{(d-r-1)}$ and obtain the graph G of Fig. 2. Then, $rad(G) = r$ and $diam(G) = d$. Moreover, it is evident that the set $S = \{v_{(r+1)}, u_{(d-r)}, w_1, w_2, w_3, \dots, w_{(k-2)}\}$ is a geodetic set and $\ddot{d}g_e$-set of G and so G has the desired properties.

Theorem 19 *For any two positive integers* a, b *with* $a \geq b + 1$ *and* $b > 2$ *there exists a connected graph with* $|V(G)| = a$, $\ddot{d}g_e(G) = b$.

Proof. Let $P: u_0, u_1, \dots, u_{(a-b)}$ be a path. Consider the graph G constructed from P by joining $b - 1$ new vertices to u_0. The graph G is a tree of order a, with b leaves. Then, $\ddot{d}g_e(G) = b$.

Theorem 20 *Let G be a graph satisfying the following two conditions:*
(i) $g_e(G) = 2$.

(ii) Let $\{u, v\}$ be the geodetic set of G. For every $u - v$ geodesic say $(u, x_1, x_2, \dots, x_{(d-1)}, v)$ there exist another distinct $u - v$ geodesic say $(u, y_1, y_2, \dots, y_{(d-1)}, v)$ such that $x_i \neq y_i, x_j = y_j$ and $x_k \neq y_k$ for $1 \leq i < j < k \leq d - 1$. Then, $\ddot{d}g_e(G) = 2$.

Proof. Let $P_x(u - v)$ be the $u - v$ geodesic $(u, x_1, x_2, \dots, x_{(D-1)}, v)$ and $P_y(u - v)$ be the $u - v$ geodesic $(u, y_1, y_2, \dots, y_{(D-1)}, v)$.
By (ii) we have, $(u, x_1, x_2, \dots, x_i, x_j, x_k, \dots, x_{(D-1)}, v)$ and $(u, y_1, y_2, \dots, y_i, y_j, y_k, \dots, y_{(D-1)}, v)$ are also geodesics between u and v. Thus all the edges of $P_x(u - v)$ and $P_y(u - v)$ lie on atleast two distinct geodesics. Since the above argument holds good for every $u - v$ geodesics we have, $\ddot{d}g_e(G) = 2$.

Theorem 21 *Let G be a graph satisfying the following two conditions:*
(i) $g_e(G) = 2$.
(ii) Let u, v be the geodetic set of G. For every $u - v$ geodesic say $(u, x_1, x_2, \dots, x_{(d-1)}, v)$ there exist another distinct $u - v$ geodesic say $(u, y_1, y_2, \dots, y_{(d-1)}, v)$ such that $\{x_i y_{i+1}, x_{i+1} y_i, x_{i+1} y_{i+2}, x_{i+2} y_{i+1}\} \in E(G)$ for some i. Then, $\ddot{d}g_e(G) = 2$.

Proof. Let $P_x(u - v)$ be the $u - v$ geodesic $(u, x_1, x_2, \dots, x_{(D-1)}, v)$ and $P_y(u - v)$ be the $u - v$ geodesic $(u, y_1, y_2, \dots, y_{(D-1)}, v)$.
By (ii) we have, $(u, x_1, x_2, \dots, x_i, x_{(i+1)}, , x_{(D-1)}, v)$ and $(u, y_1, y_2, \dots, y_i, y_{(i+1)}, \dots, y_{(D-1)}, v)$ are also geodesics between u and v. Thus all the edges of $P_x(u - v)$ and $P_y(u - v)$ lie on atleast two distinct geodesics. Since the above argument holds good for every $u - v$ geodesics we have, $\ddot{d}g_e(G) = 2$.

Theorem 22 *If* n, d *and* k *are integers such that* $3 \leq d < n, 3 \leq k < n$ *and* $n - d - k + 1 > 0$, *then there exists a graph G of order* n, *diam* $G = d$ *and* $g(G) = \ddot{d}g_e(G) = k$.

Proof. Let $P_d: u_0, u_1, u_2, \dots, u_d$ be a path of order $d + 1$. Add $k - 3$ new vertices $v_1, v_2, v_3, \dots, v_{k-3}$ to P_d by joining each vertices $v_i, 1 \leq i \leq k - 3$ to the vertex u_1, and then add a vertex x to u_{d-1} producing a tree T. Now, add $n - d - k + 1$ new vertices $w_1, w_2, w_3, \dots, w_{n-d-k+1}$ to T by joining each vertices $w_i, 1 \leq i \leq n - d - k + 1$ to both u_0 and u_2, obtaining the graph G. Then, G has order n and diameter d. Moreover, it is evident that the set $= \{u_0, u_d, x, v_1, v_2, v_3, \dots, v_{(k-3)}\}$is a geodetic set $\ddot{d}g_e$-set of G and so G has the desired properties.

Theorem 23 *For positive integers* r, d, p, k *such that with* $r < d \leq 2r$, $3 < p \leq k$ *and* $r > 1$ *there exists a graph* G *with* $radG = r$, $diamG = d$, $g(G) = p$ *and* $\ddot{d}g_e(G) = k$.

Proof. Case i: When $r = 2$ and $d = 3$, we construct a graph G with desired properties as follows. Let $P_2: u_0, u_1, u_2$ be a path of order 3. Add $p - 2$ new vertices $v_1, v_2, v_3, \dots, v_{p-2}$ to P_2 by joining each vertices $v_i, 1 \leq i \leq p - 2$ to the vertex u_1, producing a tree T. Now, add $k - p + 2$ new vertices$w_1, w_2, w_3, \dots, w_{k-p+2}$ to T by joining each vertices $w_i, 1 \leq i \leq k - p + 2$ to both u_0 and u_2, obtaining the graph$G$. Then, G has radius 2 and diameter 3.
Clearly, the geodetic set and doubly edge geodetic set of G are $S = \{u_0, u_2, v_1, v_2, v_3, \dots, v_{p-2}\}$ and $S' = \{v_1, v_2, v_3, \dots, v_{p-2}, w_1, w_2, w_3, \dots, w_{k-p+2}\}$ respectively. Thus $g(G) = p$ and $\ddot{d}g_e(G) = k$..
Case ii: When $r = 2$ and $d = 4$, we construct a graph G with desired properties as follows. Let $K_2^-: x, y$ be the complement of K_2. Add $p - 2$ new vertices $u_0, u_1, u_2, \dots, u_{p-2}$ to K_2^- by joining each vertices $u_i, 1 \leq i \leq p - 2$ to both x and y and obtain the graph H. Then, add $p - 2$ new vertices $v_1, v_2, v_3, \dots, v_{p-2}$ to H by joining each vertices v_i to each vertex $u_i, 1 \leq i \leq p - 2$ and obtain the graph H'. Now, add $k - p + 2$ new vertices $w_1, w_2, w_3, \dots, w_{k-p+2}$ to H' by joining each vertices $w_i, 1 \leq i \leq k - p + 2$ to both x and y, obtaining the graph G. Then, G has radius 2 and diameter 4. Clearly, the sets $S = \{x, y, v_1, v_2, v_3, \dots, v_{p-2}$ and $S' = \{v_1, v_2, v_3, \dots, v_{p-2}, w_1, w_2, w_3, \dots, w_{k-p+2}$ are the geodetic set and doubly edge geodetic set of G with $g(G) = p$ and $\ddot{d}g_e(G) = k$. respectively.
Case iii : When $r \geq 3$, we construct graph G with desired properties as follows.
Let $C_{2r-2}: x_1, x_2, x_3, \dots, x_{r-1}, x_r, x_{r+1}, \dots, x_{2r-2}, x_1$ be a cycle of order $2r - 2$ and let $P_{d-r}: u_0, u_1, u_2, \dots, u_{d-r}$ be a path of order $d - r + 1$. Let H be a graph obtained form $C_2 r$ and P_{d-r} by identifying x_r in C_{2r} and u_0 in P_{d-r}. Add $p - 3$ new vertices $v_1, v_2, v_3, \dots, v_{p-3}$ to H by joining each vertices $v_i, 1 \leq i \leq p - 3$ to the vertex x_1. Now, add $k - p + 2$ new vertices $w_1, w_2, w_3, \dots, w_{k-p+2}$ and join with both x_2 and x_{2r-2} and obtain the graph G. Then, $rad(G) = r$ and $diam(G) = d$. It is evident that the sets $S = \{x_2, x_{2r-2}, u_{d-r}, v_1, v_2, v_3, \dots, v_{p-3}\}$ and $S' = \{u_{d-r}, v_1, v_2, v_3, \dots, v_{p-3}, w_1, w_2, w_3, \dots, w_{k-p+2}\}$ are the geodetic set and the doubly edge geodetic set of G, with $g(G) = p$ and $\ddot{d}g_e(G) = k$., respectively

Theorem 24 *For positive integers* n, d, p *and* k *such that* $d \geq 6, 3 \leq p \leq k \leq n$, $k - p + 2 \geq 0$ *and* $n - d - k \geq 0$. *There exist a graph G of order* n, $diam(G) = d$, $g(G) = p$ *and* $\ddot{d}g_e(G) = k$.

Proof. Let $P_d: u_0, u_1, u_2, \dots, u_d$ be a path of order $d+1$. Add $p-3$ new vertices $v_1, v_2, v_3, \dots, v_{p-3}$ to P_d by joining each vertices $v_i, 1 \le i \le p-3$ to the vertex u_1, producing a tree T.Then, add $k-p+2$ new vertices $w_1, w_2, w_3, \dots, w_{k-p+2}$ to T by joining each vertices $w_i, 1 \le i \le k-p+1$ to both u_0 and u_2 obtaining the graph H. Add $n-d-k$ new vertices $x_1, x_2, \dots, x_{n-d-k}$ to H by joining each vertices $x_i, 1 \le i \le n-d-k$ to both u_4 and u_6 in H forming the graph H'. The graph G is obtainied by adding an edge $u_0 - u_3$ to H'. Then, G has order n and diameter d. Moreover, it is evident that the set $S = \{u_0, u_2, u_d, v_1, v_2, v_3, \dots, v_{p-3}\}$ is a geodesic set with $g(G) = p$ and the set $S' = \{u_d, v_1, v_2, \dots, v_{p-3}, w_1, w_2, w_3, \dots, w_{k-p+2}\}$ is a $\ddot{d}g_e$ -set of G with $\ddot{d}g_e(G) = k$.

VI. CONCLUSION

In this paper, we have formally defined the doubly edge geodetic number of a graph,certain realization problems involving doubly edge geodetic number, and studied its properties. Furthermore, we have proved that the problem is NP-complete..

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