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# Computer Algebra System and Ancient Indian Mathematics

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***Abstract*—** Computer science and Mathematical science go hand in hand in the current ongoing scenario. Arithmetic operations are the base of any digital circuit. The present era of digitization focuses on the increment in the speed of digital circuits as well as reduction in size and power consumption; thus increasing the efficiency of the entire digital circuit. The contribution of Ancient Indian Mathematicians in this regard is of significant importance. They provided unique techniques of speedy computation in the form of Sutras. These sutras are actually algorithms.
This paper describes some of the salient features and Sutras on Fundamental Arithmetic Operations of the Ancient Indian Mathematics.

***Keywords*—** Ancient Indian Mathematics, computer Algebra System, Base, Vinculum, 10's complement

## I. INTRODUCTION

Mathematical science is the common domain of computer science and technology. The computer science and the mathematical science go hand in hand in the current ongoing scenario.
The depth of the mathematical algorithms in the form of "Sutras" [6][12] (algorithms) presented by the Ancient Indian Mathematicians has not yet been gauged and explored to its fullest extent. With these Sutras calculations can be carried out mentally and quickly. There are many advantages in using a flexible, mental system [1]. The whole system is beautifully interrelated and unified. These processes not only minimize the computational time but also reduce the otherwise cumbersome calculations of conventional Mathematics to a very simple one and expose the authenticated Philosophy of Indian Mathematics. These sutras can be proved efficient for making a parallel system of computation as they are economical both in time utilization and power consumption.
Now-a-days engineers are applying Ancient Indian Mathematics in different fields such as Computer science [6], the most growing concepts of technology like Digital Signal Processing and Artificial Intelligence [8]**,** but**,** in a discrete manner. NASA is also working on Artificial Intelligence with knowledge representation in Sanskrit.
The buried treasure in the Indian mythological statements is so wide and great that they contain absolute perfection that can propel the current science to the great height.

## II. RELATED WORK

Ancient Indian Mathematics' is used in several fields of engineering and information technology, e.g. decreasing the time taken to execute programming commands for a computer, digital signal processing operation etc.

Senapati and Bhoi in 2012 used Urdhva Triyakbhyam Sutra for designing high speed multiplier that decreases the amount of time consumption.

Fernandes and Borkar reviewed the computer architectures as well as several extended work in the area and also several state-of-art applications that take full advantage of such simple Ancient Indian Mathematical unique techniques.

NASA is also working on Artificial Intelligence with knowledge representation in Sanskrit.

## III. SALIENT FEATURES

**Base System**
Concept of Base system [3] was in Ancient Indian Mathematics. Ancient Indian Mathematicians were used Base System to reduce the computations involved. The base is considered to be the powers of 10. For example 10, 100, 1000 and so on. That power of 10 should be taken as base which is nearest to the number.
For the number more than the base, portion exceeding the base is written with a positive sign in front of the number. On the other hand, for the number less than the base, the number is subtracted from the base and the difference is written down in front of the number with a negative sign. This deviation is

directly written by using the sutra which means all from 9 and last from 10.
If the difference between the number and the base is very large, then a suitable sub-base or working base can be used. Working base is a convenient multiple of the base.

**10's Complement**
Idea of complements did not start with the computer age. Use of complement in different purpose well documented in Ancient Indian Mathematics millennia before the first electronic computers were invented.
In Ancient Indian Mathematics 10's complement directly computed from the sutra ***"Nikhilam Navatascaramam Dasatah"*** [3][4][5][7] which literally means ***All From 9 and The Last From 10.*** The idea of 2's complement in binary system is straightway application of this Sutra.

**Vinculum operations**
In this process the complement of the digits bigger than 5 is taken with a negative sign and a positive carry over to the next higher order digit. For example, 79 = 80 – 1 = $8\bar{1}$, thus 79 can be written as ($8\bar{1}$). But since the digit at the tenth place is also big its complement (10 – 8 = 2) can be taken and increase the digit previous to this by 1. Thus 79 is written as ($1\overline{21}$). If there are few digits larger than 5 appearing together, their collective complement can be written by using the ***Nikhilam Sutra*** and increasing the previous digit by 1.
Thus 4798 can be written as $5\overline{202}$
And a vinculum number 4 3 4 2 can be written as $3\overline{658}$. [3][4][5]

## IV. METHODOLOGY

### Addition, Subtraction, Tables and Multiplication

### Addition
The process of addition in Ancient Indian method is converted to a sequence of simple additions, each number being always less than ten. This method is termed as ***Shuddhikaran*** [5] method in Vedic mathematics. The addition is carried out column by column in the usual manner, moving from bottom to top. Note that the addition process can be carried out in the downward direction with equal ease and accuracy.

The following example illustrates how the sum of square are carried out using the sutra ***Vertically and Crosswise***[3].

**Example 1.** To find : $321^2 + 541^2 + 234^2$

| | | |
|---|---|---|
| 3 | 2 | [1] |
| 5 | 4 | [1] |
| 2 | 3 | [4] |

**Step 1.** Unit place $1^2 + 1^2 + 4^2 = 18$

| | | |
|---|---|---|
| 3 | [2 | 1] |
| 5 | [4 | 1] |
| 2 | [3 | 4] |

**Step 2.** Ten's place
$2\{(2 \times 1) + (4 \times 1) + (3 \times 4)\} = 36$

| | | |
|---|---|---|
| [3 | 2 | 1] |
| [5 | 4 | 1] |
| [2 | 3 | 4] |

**Step 3.** Hundred's place
$2\{(3\times1)+(5\times1) + (2\times4)\}+2^2 +4^2 +3^2 = 61$

| | | |
|---|---|---|
| [3 | 2] | 1 |
| [5 | 4] | 1 |
| [2 | 3] | 4 |

**Step 4.** Thousand's place
$2\{(3 \times 2) + (5 \times 4) + (2 \times 3)\} = 64$

| | | |
|---|---|---|
| [3] | 2 | 1 |
| [5] | 4 | 1 |
| [2] | 3 | 4 |

**Step 5.** Ten Thousand's place
$3^2 + 5^2 + 2^2 = 38$

← Placement Value →

| 100,000 | 100,00 | 1000 | 100 | 10 | 1 |
|---|---|---|---|---|---|
| | | | | 1 | 8 |
| | | | 3 | 6 | |
| | | 6 | 1 | | |
| | 6 | 4 | | | |
| 3 | 8 | | | | |
| 4 | 5 | 0 | 4 | 7 | 8 |

Required answer is **4 5 0 4 7 8.**

**Application in Binary System**

**Example 2.** To find : $111^2 + 101^2 + 110^2$

| | | |
|---|---|---|
| 1 | 1 | [1] |
| 1 | 0 | [1] |
| 1 | 1 | [0] |

**Step 1.** Unit place
$1^2 + 1^2 + 0^2 = 10$

| | | |
|---|---|---|
| 1 | [1 | 1] |
| 1 | [0 | 1] |
| 1 | [1 | 0] |

**Step 2.** Ten's place
$10\{(1 \times 1) + (0 \times 1) + (1 \times 0)\} = 10$

| | | |
|---|---|---|
| [1 | 1 | 1] |
| [1 | 0 | 1] |
| [1 | 1 | 0] |

**Step 3.** Hundred's place
$10\{(1 \times 1) + (1 \times 1) + (1 \times 0)\} + 1^2 + 0^2 + 1^2 = 110$

| | | |
|---|---|---|
| [1 | 1] | 1 |
| [1 | 0] | 1 |
| [1 | 1] | 0 |

**Step 4.** Thousand's place
$10\{(1 \times 1) + (1 \times 0) + (1 \times 1)\} = 100$

| | | |
|---|---|---|
| [1] | 1 | 1 |
| [1] | 0 | 1 |
| [1] | 1 | 0 |

**Step 5.** Ten Thousand's place
$1^2 + 1^2 + 1^2 = 11$

← Placement Value →

| 128 | 64 | 32 | 16 | 8 | 2 | 1 |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | 1 | 0 |
| | | | | 1 | 0 | |
| | | | 1 | 1 | 0 | |
| | | 1 | 0 | 0 | | |
| | 1 | 1 | | | | |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 |

Required answer is **1101110.**

**Subtraction**

The most obvious problem of borrowing numbers from the left when performing subtraction can be solved very easily using the Sutra ***Nikhilam Navatascaramam Dasatah*.** Idea of subtraction was not there in Ancient Indian Mathematics. Instead of subtraction 10's complement of the subtrahend was added to the number which we follow for addition of two binary numbers.

The **Modus Operandi** of this method is **:**

Suppose b is subtracted from a

Then a - b becomes a - ($10^n$ - d), where d is the 10's complement of b with respect to $10^n$. This is obviously the same as a + d – $10^n$. Thus difficult subtraction problems become easy addition problems followed by the subtraction of powers of 10

Subtraction can be done by ***Suddhikaran*** method also.

**Tables**

Table of any number can be found using ***Nikhilam Sutra*** and ***Vinculum operation*** [3][4][5].

**Example:** Find the table of 3 9. Using vinculum 39 can be written as 41. Here the digits 4 and 1 are called operators, in writing table of 39 we take the operator 1 i.e. – 1 for the unit's place and operator **+** 4 for the ten's place and get the following table.

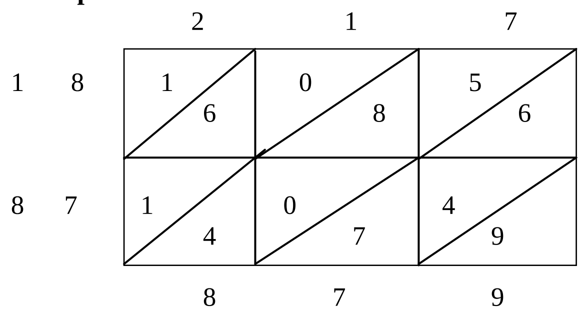
**Table of 39**

| **Level** | | **4** | $\bar{1}$ | **(operators)** |
|---|---|---|---|---|
| 1 | | 3 | 9 | |
| 2 | | 7 | 8 | |
| 3 | 1 | 1 | 7 | |
| 4 | 1 | 5 | 6 | |
| 5 | 1 | 9 | 5 | |
| 6 | 2 | 3 | 4 | |
| 7 | 2 | 7 | 3 | |
| 8 | 3 | 1 | 2 | |
| 9 | 3 | 5 | 1 | |
| 10 | 3 | 9 | 0 | |

**Multiplication**

Several Sutras and Sub Sutras are there in Ancient Indian Mmathematics to perform multiplication. Several engineering works have been performed using the Sutra ***Nikhilam Navatascaramam Dasatah,*** two Sub Sutras ***Ekadhikina Purvena*** [3][4] and ***Ekanyunena Purvena*** of ***Nikhilam Navatascaramam Dasatah*** and the Sutra ***Urdhva Tiryagbhyam*** [3][4]. These works have well documented the efficiency of these Sutras in terms of area, power, and speed.\ Following is also an alternative method of Multiplication by ***Urdhva tiryakbhyam*** given by Bhaskaracharyya in his book Lilavati.

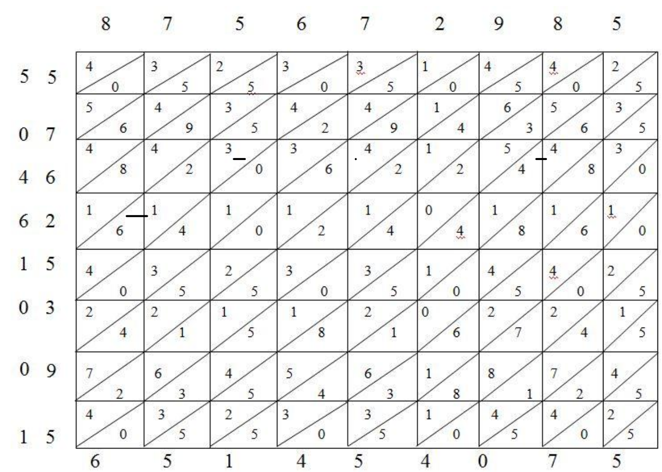
**Example:** 217 × 87




Fill the (*i*,*j*) the square by the product of the *i*th digit of the multiplicand to the *j*th digit of the multiplier. If the product is of single digit say 6 it is taken as 06.

The result is 18879.

**Example:** Find 8756729.85 × 57625.395




The product is 50 46 100 165 14. 54 07 5 (Ans.)

**N.B**. A simple calculating device (except the computer or like this) the answer cannot be given.

**The First Application (Product in Exponential Form):** No doubts the result may take a few errors. It can be minimized up to desired degree of accuracy.

**The Second Application (Product in Various Base Systems) :**

Product of two nos. with a given base can be performed and get the answer in the same base.

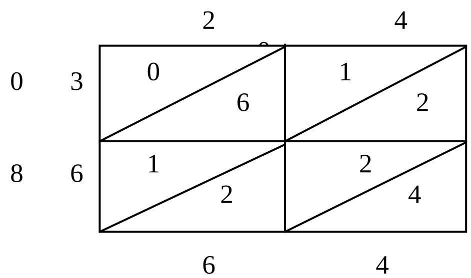
 

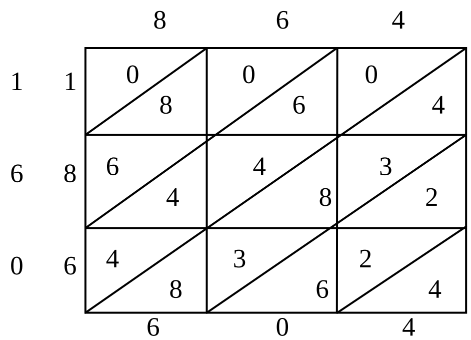
**The Third Application (Square of a Large Number) :** The square of any number can be found in three lines only by taking half of the table.

**First Application:**

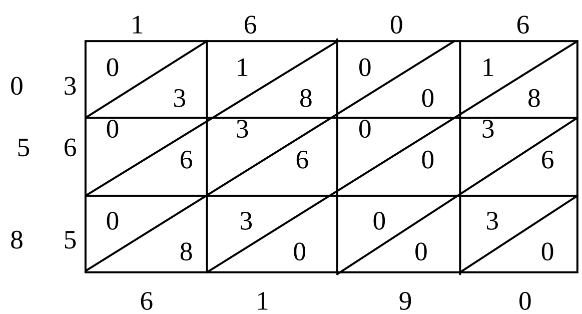
**Example:** Find the value of a light year in miles.

Assuming the velocity of light as 186000 mile / s, the distance travel by light in one year = 186000 × 3600 × 24 × 365 mile = 24 × 36 ×186 × 365 ×$10^5$ mile.




1 6 0 6 0 4 = 1606 × $10^2$

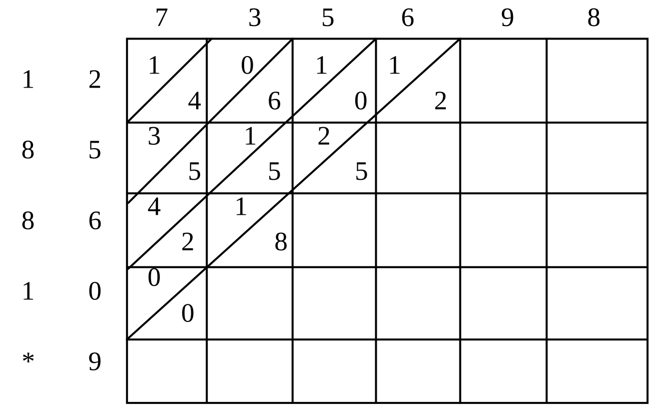



5 8 6 1 9 0 = 5.86 × $10^5$ mile

.: 24 × 36 ×186 × 365 ×$10^5$ mile = 5.86 × $10^{10}$ mile

**Example:** Find 735698 × 25609 by rounding off in two / three significant figure.

At first, roughly four significant figures will be found and then the answer is rounded off to desired no. of accuracy.




.: 735698 × 256 09

= 1881 × $10^7$

= 1.9 × $10^{10}$ (two significant figure)

= 1.88 × $10^{10}$ (in three significant figure)

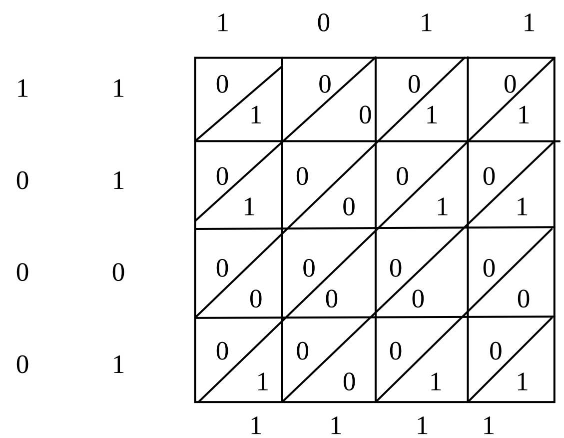
N.B. : The actual answer is 188 40 49 00 82

If two significant figure is taken the error is = - 15 95 099 18 i.e., -1.5 ×$10^8$

and if three significant figure is taken the error is = 40 49 00 82 = 0.04 ×$10^8$

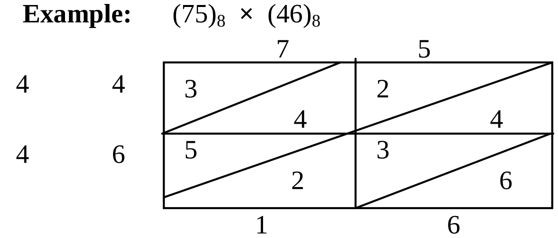
**Second Application:**

**Example**: $(1011)_2$ × $(1101)_2$




.: $(1011)_2$ × $(1101)_2$ = $(10001111)_2$

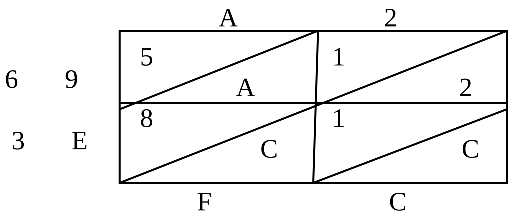
**Example:** $(75)_8$ × $(46)_8$




.: $(75)_8$ × $(46)_8$ = $(44\ 16)_8$

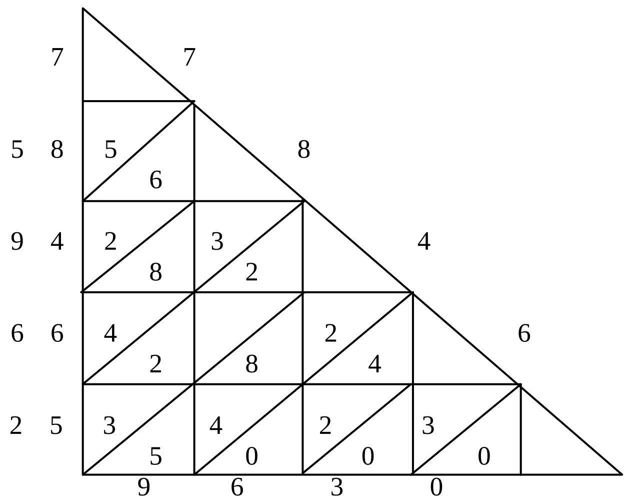
**Example: A2 × 9 E**




.: $(A2)_{16}$ × $(9E)_{16}$ = $(63\ FC)_{16}$

**Third Application:**

**Example: $78465^2$**




5 9 6 2 9 6 3 0 → Half portion
5 9 6 2 9 6 3 0 → Half portion
4 9 6 4 1 6 3 6 2 5 → The diagonal

6 1 5 6 7 5 6 2 2 5

## V. CONCLUSION

This paper focuses on the salient features of Ancient Indian Mathematics and how those are used in some of the fundamental operations of Arithmetic. Arithmetic operations are the basis of any digital circuit. The techniques are just a part of a complete system of Mathematics. Several engineers have started to use these techniques in different fields like digital signal processing, image processing, artificial intelligence etc. and have got better results. More focus has to be given not only on these techniques but also other parts like Sulva Sutra, Pingala etc. I think there is a lot of scope of applying our vast treasure in computer field.

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