IJCSE International Journal of Computer Sciences and Engineering Open Access
Research Paper Vol-6, Special Issue-4, May 2018 E-ISSN: 2347-2693

Application of Chebyshev Neural Network for Function Approximation

M.Sornam[1*], V.Vanitha[2]

[1*,2] Dept. of Computer Science, University of Madras, Chennai, India

**Corresponding Author: madasamy.sornam@gmail.com, Tel.: 9841166535*

Available online at: www.ijcseonline.org

Abstract— Function Approximation is a major need in many areas such as Applied Mathematics, Computer Science, Engineering problems and so on. This paper proposed a solution for performing function approximation by using novel functional Chebyshev Neural Network with Backpropagation Algorithm. The advantage of Chebyshev Neural Network is very efficient for computation because of less complexity in modelling of the structure and produces the fast convergence rate and it is easy to implement circuit implementation compared to the standard Multilayer feed forward neural network. The proposed network consists of single input and a single output. The hidden layer is designed as taking the input of numerically transformable Chebyshev polynomial expansion of input. Backpropagation algorithm with Chebyshev Neural Network shows good behaviour in Nonlinear Function Approximation compared to multilayer feed forward neural network. The performance metric used in this paper to compare the realization capability of two networks for training and testing phase is Mean Square Error.

Keywords— Function Approximation, Chebyshev Neural Network, Multilayer Perceptron, Backpropagation Algorithm

I. INTRODUCTION

There is a wide range of Neural Network applications in classification, prediction and clustering problems and neural network shows it efficiency in those areas. Nowadays there is a growing interest in the field of the nonlinear system. Many research works have been done to study the approximation capacity of the neural network. The superior network of Backpropagation Neural Network is Radial basis function network which is good in approximation capacity and the performance of the network based on finding the central position but, there is no proper guideline to determine the central position, it was identified only based on experience. Hence it is very difficult to predict the performance of the RBFN. It was already well proven that the Multi-Layer Perception with Backpropagation algorithm is very robust to perform a complex task in a nonlinear environment but the drawback is the computational complexity of the network and the backpropagation algorithm is trapped with local minima. When we use the two existing aforementioned network for nonlinear function approximation, there is a contradiction will occur on between the network complexity and approximation accuracy. These problems overcome and resolved by the Chebyshev Neural network and the activation function of the neuron in the hidden layer is the group of Chebyshev orthogonal polynomials.

The research work of S.P.Yan et all used the CNN in chemistry areas and show it is the best approximation theory [1]. CNN is widely used in chaos systems [3][8], discrete time nonlinear systems [2] and aerospace and spacecraft [10] applications. CNN also shows its best performance in the field of image processing. The work of sudhansu Kumar Mishra et al [4] and Sornam et al [11], proposed CNN to denoise the image corrupted by a different type of noises and shows CNN is very efficient for noise removal. CNN work well in numerical solution problem for example emben-fowler type equation [9]. The FLANN network has been used for the function expansion of sine and cosine function for the problem of nonlinear dynamic system reported by Patra and his co-authors which reveals the functional expansion based neural network work well in approximation based problems [5]. The primary purpose of this paper is to highlight the efficiency of the Chebyshev Neural Network in the problem of nonlinear function approximation. The CNN based network produce the better accuracy and fast convergence rate with the backpropagation algorithm compared with the MLP.

The rest of the paper is organized as follows, Section II discuss the architecture and advantage of Chebyshev Neural Network. The algorithm used for making the network to realize the nonlinear function explained in Section III. The Section IV illustrated the Experimental result. The section V contains the conclusion followed by reference.

II. CHEBYSHEV NEURAL NETWORK

The structure of Artificial Neural Network proposed in this paper is Chebyshev Neural Network. It is a function link network based on Chebyshev polynomials. The architecture of CNN consists of two parts. They are numerical transformation part and learning part. Numerical transformation part is used for giving the input to the hidden layer by expanding the input as a finite number of Chebyshev polynomials of first order. The result of the Chebyshev polynomial can be viewed as new input vector. The Chebyshev polynomial of the first kind $T_n(z)$ can be defined by the contour integral function

$$T_n(z) = \frac{1}{4\pi i} \oint \frac{(1-t^2)\, t^{-n-1}}{(1-2tz+t^2)}\, dt, \qquad (1)$$

The Architecture of Chebyshev neural network consist of the input layer, hidden layer and output layer. The Weights between the input layer and hidden layer are set to 1 and the weights from hidden to output are randomly generated in the range of -1 to 1. The hidden layer has 4 neurons and input to the hidden layer are the transfer functions which is a group of Chebyshev orthogonal polynomial functions $T_n(X)$ where X is the input [6].

The Chebyshev polynomials can be generated by using the following recursive formula is

$$T_{n+1}(X) = 2XT_n(X) - T_{n-1}(X) \qquad (2)$$

The first few Chebyshev Polynomial of the first kind is

$$\left.\begin{array}{l} T_0(X) = 1 \\ T_1(X) = X \\ T_2(X) = 2X^2 - 1 \\ T_3(X) = 4X^3 - 3X \end{array}\right\} \qquad (3)$$

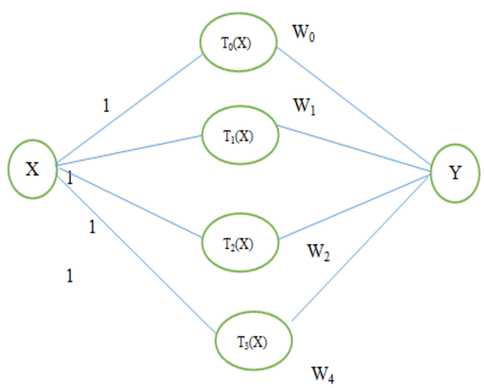


Figure 1. Architecture of CNN

III. APPROXIMATION ALGORITHM FOR CNN

Backpropagation is a supervised learning algorithm for training the Artificial neural network to calculate the gradient descent for updating the weights to make the network to produce the desired output. It is very robust algorithm and its work based on the principle of backward propagation of error, hence the error is calculated at the output and passes back through the network layers. In recent years, this algorithm is widely used in deep learning architecture for training phase.

The Activation function used to fire the neuron is Sigmoid Activation function. The sigmoid function is a transform function mathematically called as S- shaped curve used to find the output of the neuron. It maps the output value between 0 and 1. The Proposed network trained by using the following procedure.

Set the Chebyshev neural network as follows

Input layer: S = X

Input of hidden layer neuron is $Net_i = S$

Output of hidden layer neuron is $Out_i = T_i(Net_i)$

Output layer:

$$Y = \sum W_i T_i(X) \qquad (4)$$

Sigmoid Activation function for the output layer is

$$F(x) = 1 / 1 + e^{-x} \qquad (5)$$

The Error is calculated using the formula

$$E = f(X_t) - Y_t \qquad (6)$$

In this proposed network there is no need to do the weight updation between the hidden and input layer. To converge the network to produce the desired output, the weight updation can be done only between the hidden and output layer by using the following formula

$$deltaW_i = (Output_i - Target_i) * Output_i(1 - Output_i) \qquad (7)$$

Adjust the weights,

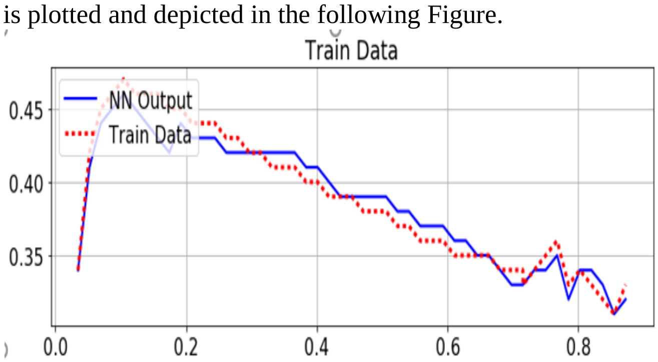
$$Weights_i = Weights_i + deltaW_i \qquad (8)$$

IV. EXPERIMENTAL RESULTS

The Experiment was done to realize the nonlinear function $Y = 0.5 * Tanh(25*V) - Tanh(V) + 0.2 * Tanh(V) + 0.03 * V$ and approximation done using Chebyshev neural network with Backpropagation and MLP with Backpropagation. The result shows CNN based network produce better accuracy and give fast convergence rate compare to standard MLP with Backpropagation.

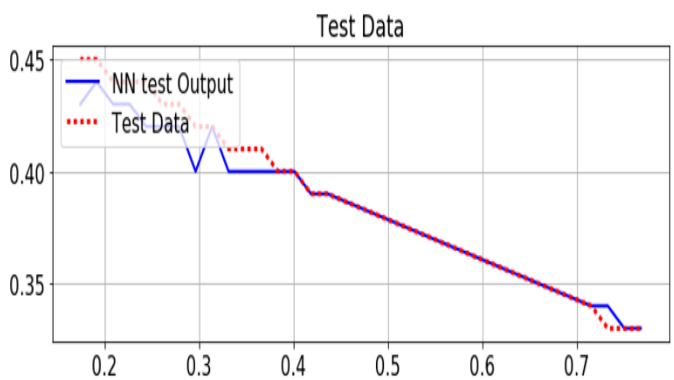
A. Chebyshev Neural Network with BP Algorithm

The Network consists of 1 x 4 x 1 structure as shown in Figure 1. The number of samples for training the network is 50. The learning rate set as 0.5. Number of epochs taken for the network to realize the function is 200 and produce the mean square error as E = 1.0 x 10^{-3}. The weight between the hidden and output are constantly updated in each and every epoch during the learning process to meet the requirements of function approximation. The final approximation curve produced by the network along with the actual output curve is plotted and depicted in the following Figure.



X- Axis – Input; Y-axis - Output

Figure 2. CNN Plot for Training Dataset

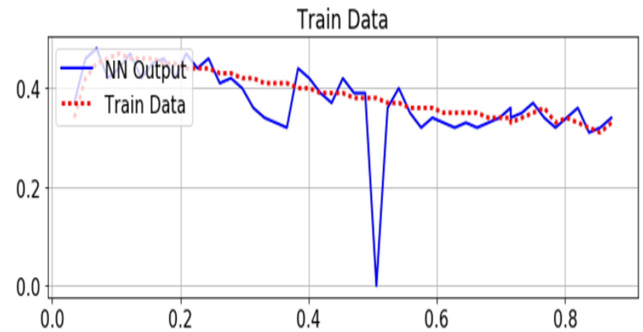


X- Axis – Input; Y-axis - Output

Figure 4. CNN Plot for Testing Dataset

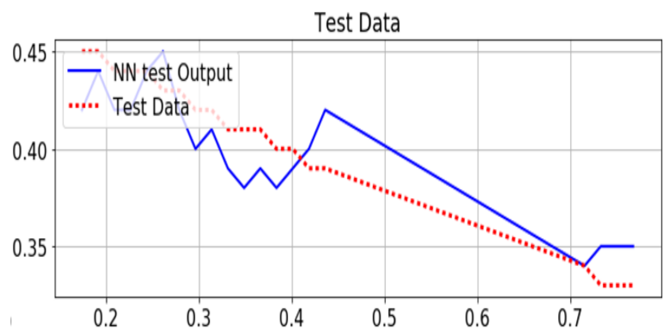
B. Mutlilayer Network with BP Algorithm

The Network consists of 1 x 8 x 8 x 1. The number of samples for training the network is 50. The learning rate set as 0.5. Number of epochs taken for the network to realize the function is 1000 and produce the mean square error as E = 1.0 x 10^{-2}. The weight between the input layer and hidden layer as well as the weight between hidden and output are constantly updated in each and every epoch during the learning process to meet the requirements of function approximation. The final approximation curve produced by the network along with the actual output curve is plotted and depicted in the following Figure



X- Axis – Input; Y-axis - Output

Figure 3. MLP Plot for Training Dataset



X- Axis – Input; Y-axis - Output

Figure 5. MLP Plot for Testing Dataset

The following table summarize the results of the experiment.

Table 1. Results of Function Approximation

S.No	**Network**	**No. of Samples**	**No. of Epochs**	**Mean Square Error**
1	CNN (Training Data)	50	200	6 x 10^{-3}
2	CNN (Testing Data)	20	1	8 x 10^{-3}
3	MLP (Training Data)	50	1000	1 x 10^{-2}
4	MLP (Testing Data)	20	1	5 x 10^{-2}

V. CONCLUSION

In this paper, the nonlinear function approximation has been recognized using two different Artificial Neural Networks namely Chebyshev based network and Multilayer Perceptron feed forward network. The result shows that Chebyshev based network produces more accuracy and fast convergence rate compare to MLP. The advantage of Chebyshev based network is since the network have fixed structure, it can be implemented easily and the computation complexity also drastically reduced. The result concludes that the proposed network is five times faster than standard feed forward neural network. Chebyshev based network are hopeful to be used in future in the aspects of data compression, Pattern recognition, image processing and engineering applications to get an optimum solution.

References

[1] Yan, S. P., et al. "*CO2 concentration detection based on Chebyshev neural network and best approximation theory.*" Instrument Technique and Sensor 6 (2011): 107-110.

[2] Shrivastava, Animesh Kumar, and Shubhi Purwar. "*State feedback and output feedback tracking control of discrete-time nonlinear system using Chebyshev neural networks*." Power, Control and Embedded Systems (ICPCES), 2010 International Conference on. IEEE, 2010.

[3] Zou, An-Min, Krishna Dev Kumar, and Zeng-Guang Hou. "*Quaternion-based adaptive output feedback attitude control of spacecraft using Chebyshev neural networks*." IEEE transactions on neural networks 21.9 (2010): 1457-1471.

[4] Mishra, Sudhansu Kumar, Ganpati Panda, and Sukadev Meher. "*Chebyshev functional link artificial neural networks for denoising of image corrupted by salt and pepper noise*." (2009).

[5] Patra, Jagdish Chandra, et al. "*Identification of nonlinear dynamic systems using functional link artificial neural networks.*" IEEE Transactions on Systems, Man, and Cybernetics, Part B *(Cybernetics)* 29.2 (1999): 254-262.

[6] Wang, Lidan, Meitao Duan, and Shukai Duan. "*Memristive chebyshev neural network and its applications in function approximation*." Mathematical Problems in Engineering 2013 (2013).

[7] Purwar, Shubhi, Indra Narayan Kar, and Amar Nath Jha. "*On-line system identification of complex systems using Chebyshev neural networks*." Applied soft computing 7.1 (2007): 364-372.

[8] Akritas, P., I. Antoniou, and V. V. Ivanov. *"Identification and prediction of discrete chaotic maps applying a Chebyshev neural network."* Chaos, Solitons & Fractals 11.1-3 (2000): 337-344.

[9] Mall, Susmita, and Snehashish Chakraverty. "*Numerical solution of nonlinear singular initial value problems of Emden–Fowler type using Chebyshev Neural Network method*." Neurocomputing 149 (2015): 975-982.

[10] Zou, An-Min, Krishna Dev Kumar, and Zeng-Guang Hou. "*Quaternion-based adaptive output feedback attitude control of spacecraft using Chebyshev neural networks*." IEEE transactions on neural networks 21.9 (2010): 1457-1471.

[11] Sornam, Madasamy, V. Vanitha, and T. G. Ashmitha. "*Noise Removal using Chebyshev Functional Link Artificial Neural Network with Back propagation.*" International Journal of Advanced Research in Computer Science 8.5 (2017).

Authors Profile

Dr M. Sornam received her MSc in Mathematics from the University of Madras in the year 1987, Master’s Degree in Computer Applications from the University of Madras in the year 1991 and received her Ph.D from the University of Madras in the year 2013. Since 1991–1996, she worked as a Lecturer in Computer Science at Anna Adarsh College, Chennai. Later, from 1996 to 2000 she worked as a Lecturer in Computer Science at T.S. Narayanasami College of Arts and Science, Chennai. Since 2001, she has been working in the Department of Computer Science, University of Madras. At present, she is working as an Associate Professor in Computer Science at the University of Madras. Her area of interest includes artificial intelligence and artificial neural networks , image processing, data mining, pattern recognition and applications.

Mrs. V.Vanitha received her M.Phil Degree in Computer Science, from University of Madras, Chennai in the year 2012. She is currently pursuing Ph.D. at University of Madras.She has completed her under graduate in Computer science in Sri Meenakshi Govt college for Women, Madurai and Post Graduate M.Sc Computer Science in Madurai Kamaraj University. At present, She is working as Assisant Professor in the Department of Computer Applications at Ethiraj College for Women, Chennai, since 2012. Her research area includes artificial intelligence, artificial neural networks, image processing and pattern recognition.

